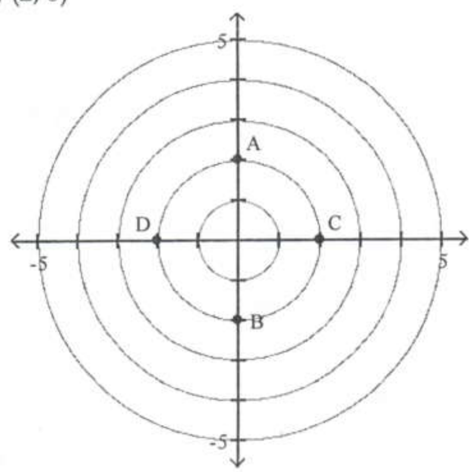
Name Key

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Match the point in polar coordinates with either A, B, C, or D on the graph.

1) (2, 0) — 1) C



Find another representation, (r, θ), for the point under the given conditions.

2) $\left(1, \frac{\pi}{4}\right)$, $r < 0$ and $0 < \theta < 2\pi$ — neg

$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

2) $\left(-1, \frac{5\pi}{4}\right)$

3) $\left(3, \frac{\pi}{3}\right)$, $r > 0$ and $-2\pi < \theta < 0$ — pos, neg

$\theta = \frac{\pi}{3} - 2\pi = -\frac{5}{3}\pi$

3) $\left(3, -\frac{5}{3}\pi\right)$

Polar coordinates of a point are given. Find the rectangular coordinates of the point.

4) (-9, 120°)

$x = r\cos\theta = -9\cos 120 = -9\left(-\frac{1}{2}\right) = 4.5$

$y = r\sin\theta = -9\sin 120 = -9\left(\frac{\sqrt{3}}{2}\right) = -\frac{9}{2}\sqrt{3}$

4) $\left(\frac{9}{2}, -\frac{9\sqrt{3}}{2}\right)$

5) $\left(7, \frac{3\pi}{4}\right)$

$x = r\cos\theta = 7\cos\frac{3\pi}{4} = 7\left(-\frac{\sqrt{2}}{2}\right) = -\frac{7\sqrt{2}}{2}$

$y = r\sin\theta = 7\sin\frac{3\pi}{4} = 7\left(\frac{\sqrt{2}}{2}\right) = \frac{7\sqrt{2}}{2}$

5) $\left(-\frac{7\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}\right)$

The rectangular coordinates of a point are given. Find polar coordinates of the point. Express θ in radians.

6) $(3, -3\sqrt{3})$

$r = \sqrt{x^2+y^2} = \sqrt{3^2 + (-3\sqrt{3})^2} = \sqrt{9 + 27} = \sqrt{36} = 6$

$\theta = \tan^{-1}\left(-\frac{3\sqrt{3}}{3}\right) = 300° = \frac{5}{3}\pi$

6) $\left(6, \frac{5\pi}{3}\right)$

7) (-5, 0) — 7) $(5, \pi)$

Convert the rectangular equation to a polar equation that expresses r in terms of θ.

8) $x = 9$

$r\cos\theta = 9$

$r = \frac{9}{\cos\theta}$

8) $r = \frac{9}{\cos\theta}$

9) $6x - 9y + 10 = 0$

$6r\cos\theta - 9r\sin\theta + 10 = 0$

$r(6\cos\theta - 9\sin\theta) = -10$

$r = \frac{-10}{6\cos\theta - 9\sin\theta}$

9) $r = \frac{-10}{6\cos\theta - 9\sin\theta}$

Convert the polar equation to a rectangular equation.

10) $r = 8\csc\theta$

$r = \frac{8}{\sin\theta}$

$r\sin\theta = 8$

$\cancel{x} = 8$

10) $y = 8$

$$x^2+y^2=5x+2y$$
$$x^2-5x \quad +y^2-2y \quad =0$$
$$x^2-5x+\frac{25}{4}+y^2-2y+1=\frac{25}{4}+1$$
$$\left(x-\frac{5}{2}\right)^2+(y-1)^2=\frac{29}{4}$$

11) $r = 5\cos\theta + 2\sin\theta$

$r^2 = 5r\cos\theta + 2r\sin\theta$

11) $x^2+y^2=5x+2y$

Find the absolute value of the complex number.

12) $z = 4 + 10i$

$|z| = \sqrt{4^2+10^2} = \sqrt{116}$ $\quad 2 \cdot 58 \quad 2 \cdot 29 \quad 2\sqrt{29}$

12) $2\sqrt{29}$

Write the complex number in polar form. Express the argument in degrees.

13) $-2i$

$0 + -2i$ (a, b)

$r = \sqrt{0^2+(-2)^2} = 2$

$\theta = \frac{3\pi}{2}$ or $270°$

13) $2\text{cis}\,270°$

Write the complex number in polar form. Express the argument in radians.

14) $-4\sqrt{3} - 4i$

$-4\sqrt{3}$, -4, 30, 8

$\pi + \frac{\pi}{6}$

$r = 8$

$\sqrt{(-4\sqrt{3})^2+(4)^2} = \sqrt{48+16} = \sqrt{64} = 8$

14) $8\,\text{cis}\,\frac{7\pi}{6}$

Write the complex number in rectangular form.

15) $-3(\cos 225° + i \sin 225°)$

r, θ

$x = r\cos\theta = -3\cos 225 = -3\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\sqrt{2}}{2}$

$y = r\sin\theta = -3\sin 225 = -3\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\sqrt{2}}{2}$

15) $\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

16) $-3\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

r, θ

$\sqrt{3}$, 2, 1

$x = r\cos\theta = -3\cos\frac{2\pi}{3} = -3\left(-\frac{1}{2}\right) = \frac{3}{2}$

$y = r\sin\theta = -3\sin\frac{2\pi}{3} = -3\left(\frac{\sqrt{3}}{2}\right) = -\frac{3\sqrt{3}}{2}$

16) $\frac{3}{2} - \frac{3\sqrt{3}}{2}i$

Find the product of the complex numbers. Leave answer in polar form.

17) $z_1 = 5(\cos 20° + i \sin 20°)$
$z_2 = 4(\cos 10° + i \sin 10°)$

$z_1 z_2 = 5 \cdot 4\,\text{cis}(20+10) = 20\,\text{cis}\,30$

17) $20\,\text{cis}\,30°$

18) $z_1 = 4i$
$z_2 = -6 + 6i$

$4\,\text{cis}\,90$

$6\sqrt{2}\,\text{cis}\,135$

$z_1 z_2 = 4(6\sqrt{2})\,\text{cis}(90+135)$
$= 24\sqrt{2}\,\text{cis}\,225$ or $\frac{5\pi}{4}$

18) $24\sqrt{2}\,\text{cis}\,\frac{5\pi}{4}$

Find the quotient $\frac{z_1}{z_2}$ of the complex numbers. Leave answer in polar form.

19) $z_1 = 8\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$
$z_2 = 3\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

$\frac{z_1}{z_2} = \frac{8}{3}\,\text{cis}\left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \frac{8}{3}\,\text{cis}\,\frac{\pi}{3}$

19) $\frac{8}{3}\,\text{cis}\,\frac{\pi}{3}$

20) $z_1 = 4i$
$z_2 = -6 + 6i$

$\frac{4}{6\sqrt{2}}\,\text{cis}(90-135) = \frac{4\sqrt{2}}{12}\,\text{cis}(-45) = \frac{\sqrt{2}}{3}\,\text{cis}\,\frac{7\pi}{4}$

$-\frac{\pi}{4}$ or $\frac{7\pi}{4}$

20) $\frac{\sqrt{2}}{3}\,\text{cis}\,\frac{7\pi}{4}$

Use DeMoivre's Theorem to find the indicated power of the complex number. Write the answer in rectangular form.

21) $[2(\cos 15° + i \sin 15°)]^4$

2, 1, $\sqrt{3}$

$2^4\,\text{cis}(4(15)) = 16\,\text{cis}\,60$

$16\cos 60 + 16\sin 60\,i = 16\left(\frac{1}{2}\right) + 16\left(\frac{\sqrt{3}}{2}\right)i$

21) $8 + 8\sqrt{3}i$

22) $(-\sqrt{3} + i)^6$

1, 2, 30, $-\sqrt{3}$

$[2(\text{cis}\,150)]^6 = 2^6\cos 900 + 2^6 \sin 900\,i = -64 + 0i$

-720, 180

22) -64

Find all the complex roots. Write the answer in the indicated form.

23) The complex square roots of $4(\cos 120° + i \sin 120°)$ (polar form)

$\sqrt{4}\,\text{cis}\,\frac{120}{2} = 2\,\text{cis}\,60°$

$2\,\text{cis}\,\frac{120+360}{2}$

$2\,\text{cis}\,240$

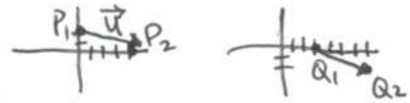
23) $2\,\text{cis}\,60$ & $2\,\text{cis}\,240$

Find the specified vector or scalar.

24) $\mathbf{u} = -6\mathbf{i} - 2\mathbf{j}$, $\mathbf{v} = 8\mathbf{i} + 7\mathbf{j}$; Find $\mathbf{u} - \mathbf{v}$.

$-6i - 2j - 8i - 7j$

24) $-14i - 9j$



MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the problem.

$u = (4-0)i + (0-2)j = 4i - 2j$ $\quad \|u\| = \sqrt{4^2 + (-2)^2} = 2\sqrt{5}$

25) Let vector **u** have initial point $P_1 = (0, 2)$ and terminal point $P_2 = (4, 0)$. Let vector **v** have initial point $Q_1 = (3, 0)$ and terminal point $Q_2 = (7, -2)$. **u** and **v** have the same direction. Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$. Is $\mathbf{u} = \mathbf{v}$? 25) D

$v = (7-3)i + (-2-0)j = 4i - 2j$ $\quad \|v\| = \dots = 2\sqrt{5}$

A) $\|\mathbf{u}\| = \sqrt{6}, \|\mathbf{v}\| = \sqrt{6}$; yes
B) $\|\mathbf{u}\| = 2\sqrt{5}, \|\mathbf{v}\| = 2\sqrt{5}$; no
C) $\|\mathbf{u}\| = 6, \|\mathbf{v}\| = 6$; no
D) $\|\mathbf{u}\| = 2\sqrt{5}, \|\mathbf{v}\| = 2\sqrt{5}$; yes

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the unit vector that has the same direction as the vector v.

26) $\mathbf{v} = 12\mathbf{i} + 5\mathbf{j}$ $\quad \dfrac{v}{\|v\|} = \dfrac{12i + 5j}{\sqrt{12^2 + 5^2}} = \dfrac{\dots}{13}$ 26) $\frac{12}{13}i + \frac{5}{13}j$

Let v be the vector from initial point P_1 to terminal point P_2. Write v in terms of i and j.

27) $P_1 = (6, 4)$; $P_2 = (-4, -5)$ $\quad v = (-4-6)i + (-5-4)j = -10i - 9j$ 27) $v = -10i - 9j$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the angle between the given vectors. Round to the nearest tenth of a degree. $\cos\theta = \dfrac{u \cdot v}{\|u\| \cdot \|v\|} = \dfrac{-2(4) + (5)(-6)}{\sqrt{29}\sqrt{52}}$

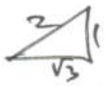
28) $\mathbf{u} = -2\mathbf{i} + 5\mathbf{j}$, $\mathbf{v} = 4\mathbf{i} - 6\mathbf{j}$ $\quad \theta = \cos^{-1}(\dots) = 168.1$ 28) C

A) 84.1° B) 178.1° C) 168.1° D) 74.1°

$\|u\| = \sqrt{(-2)^2 + (5)^2} = \sqrt{4 + 25} = \sqrt{29}$ $\quad \|v\| = \sqrt{(4)^2 + (-6)^2} = \sqrt{16 + 36} = \sqrt{52}$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Write the vector v in terms of i and j whose magnitude $\|\mathbf{v}\|$ and direction angle θ are given.



29) $\|\mathbf{v}\| = 10$, $\theta = 120°$ 29) $v = -5i + 5\sqrt{3}j$

$v = 10\cos 120\, i + 10 \sin 120\, j$
$= 10(-\frac{1}{2})i + 10(\frac{\sqrt{3}}{2})j$

Find all the complex roots. Write the answer in the indicated form.

30) The complex square roots of $2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$ (rectangular form) 30) $\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$; $-\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$

$\sqrt{2}\, \mathrm{cis}\left(\dfrac{\frac{2\pi}{3}}{2}\right) = \sqrt{2}\, \mathrm{cis}\, \frac{\pi}{3}$

$\rightarrow \sqrt{2}\cos\frac{\pi}{3} + \sqrt{2}\sin\frac{\pi}{3}\, i = \sqrt{2}(\frac{1}{2}) + \sqrt{2}(\frac{\sqrt{3}}{2})$

$\sqrt{2}\, \mathrm{cis}\left(\dfrac{\frac{2\pi}{3} + 2\pi}{2}\right)$

$\rightarrow \sqrt{2}\, \mathrm{cis}\, \frac{8\pi}{6} = \sqrt{2}\, \mathrm{cis}\, \frac{4\pi}{3}$

$\sqrt{2}\cos\frac{4\pi}{3} + \sqrt{2}\sin\frac{4\pi}{3}\, i$

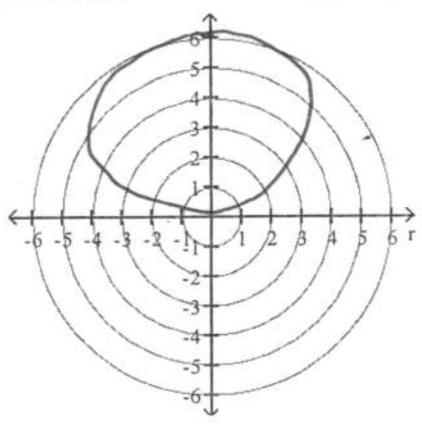
Use the given vectors to find the specified scalar.

31) $\mathbf{u} = -14\mathbf{i} + 10\mathbf{j}$ and $\mathbf{v} = 13\mathbf{i} + 10\mathbf{j}$; Find $\mathbf{u} \cdot \mathbf{v}$. 31) -82

$u \cdot v = (-14)(13) + (10)(10) = -82$

Graph the polar equation.

32) $r = 6 \sin \theta$ circle 32) ←



33) $r = 6 - \cos\theta$

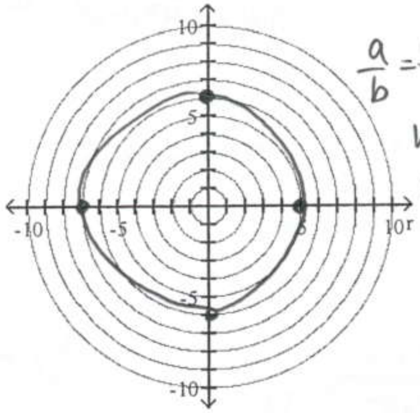
limaşon

$\frac{a}{b} = \frac{6}{1} > 2$

no dimple
no inner loop

θ	$6-\cos\theta$
0	5
$\frac{\pi}{2}$	6
π	7
$\frac{3\pi}{2}$	6

33) ←______



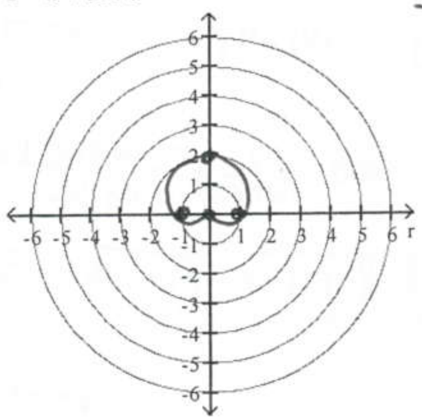
34) $r = 1 + \sin\theta$

$\frac{a}{b} = 1$

Cardioid

θ	$1+\sin\theta$
0	1
$\frac{\pi}{2}$	2
π	1
$\frac{3\pi}{2}$	0

34) ←______



35) $r^2 = 16\cos(2\theta)$

Lemniscate

$r = \sqrt{16}$
$r = 4$

35) ←______