Ch 3 Practice Test

Exam

Name Key

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Write the equation in its equivalent exponential form.

1) $\log_8 64 = x$ — 1) $8^x = 64$

2) $\log_b 64 = 3$ — 2) $b^3 = 64$

Write the equation in its equivalent logarithmic form.

3) $3^{-2} = \frac{1}{9}$ — 3) $\log_3 \frac{1}{9} = -2$

4) $a^4 = 4096$ — 4) $\log_a 4096 = 4$

Evaluate the expression without using a calculator.

5) $\log_2 8$ $\quad 2^x = 8$ — 5) $x = 3$

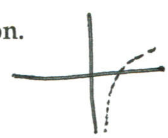
6) $\log_6 \frac{1}{6}$ $\quad 6^x = \frac{1}{6}$ — 6) $x = -1$

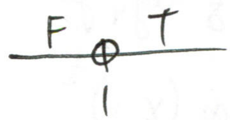
7) $\log_4 \frac{1}{\sqrt{4}}$ $\quad 4^x = \frac{1}{\sqrt{4}}$ — 7) $x = -\frac{1}{2}$

8) $\log_3 3^{11}$ $\quad 3^x = 3^{11}$ — 8) $x = 11$

Find the domain of the logarithmic function.

9) $f(x) = \log_5 (x - 1)$ $\quad x = 1$ (right 1)

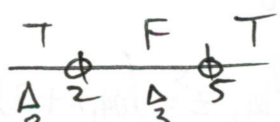




9) $(1, \infty)$

* 10) $f(x) = \log (x^2 - 7x + 10)$

$(x-5)(x-2)$ $\quad x = 2, 5$



* 10) $(-\infty, 2) \cup (5, \infty)$

Evaluate or simplify the expression without using a calculator.

11) $\log\left(\frac{1}{100}\right)$ $\quad 10^x = \frac{1}{100}$ — 11) $x = -2$

12) $10^{\log 4}$ — 12) $x = 4$

13) $10^{\log \sqrt[7]{x}}$

13) $\sqrt[7]{x}$

14) $\ln \sqrt[8]{e}$ $\quad \ln e^{\frac{1}{8}}$

14) $\frac{1}{8}$

Evaluate the expression without using a calculator.

15) $\ln e^{17x}$

15) $17x$

16) $e^{\ln 17x^3}$

16) $17x^3$

Use properties of logarithms to expand the logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

17) $\log (100x)$ $\quad \log 100 + \log x = \log 10^2 + \log x = 2\log 10 + \log x$

17) $2 + \log x$

18) $\log_5 (125x)$ $\quad \log_5 125 + \log_5 x = \log_5 5^3 + \log_5 x$

18) $3 + \log_5 x$

19) $\ln\left(\frac{e^2}{11}\right)$ $\quad \ln e^2 - \ln 11 = 2\ln e^1 - \ln 11$

19) $2 - \ln 11$

20) $\log_4\left(\frac{16}{x}\right)$ $\quad \log_4 4^2 - \log_4 x =$

20) $2 - \log_4 x$

21) $\log N^{-5}$

21) $-5\log N$

22) $\log_4 \sqrt[8]{y}$ $\quad \log_4 y^{\frac{1}{8}} = \frac{1}{8}\log_4 y$

22) $\frac{1}{8}\log_4 y$

23) $\log_4\left(\frac{16}{\sqrt{x-1}}\right)$ $\quad \log_4 16 - \log (x-1)^{\frac{1}{2}}$

23) $2 - \frac{1}{2}\log (x-1)$

24) $\log_b\left(\frac{xy^2}{z^5}\right)$ $\quad \log_b(xy^2) - \log_b z^5 = \log_b x + 2\log_b y - 5\log_b z$

24) $\log_b x + 2\log_b y - 5\log_b z$

25) $\log\left[\frac{2x^3\sqrt[3]{4-x}}{5(x+4)^2}\right]$ $\quad \log 2 + 3\log x + \frac{1}{3}\log (4-x) - \log 5 - 2\log (x+4)$

25) ______

Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

26) $(\log_a x - \log_a y) + 2\log_a z$

$\log_a \frac{xz^2}{y}$

26) $\log_a \frac{xz^2}{y}$

27) $\frac{1}{2}(\log_3 (r - 8) - \log_3 r)$

$\log_3 \frac{r-8}{r}$

27) $\log_3 \sqrt{\frac{r-8}{r}}$

28) $\frac{1}{7}[3\ln (x + 1) - \ln x - \ln (x^2 - 8)]$

$\ln \frac{(x+1)^3}{x(x^2-8)}$

28) $\ln \sqrt[7]{\frac{(x+1)^3}{x(x^2-8)}}$

Use common logarithms or natural logarithms and a calculator to evaluate to four decimal places

29) $\log_{0.6} 16$

$\frac{\log 16}{\log 0.6} \approx$

29) -5.4277

30) $\log_{\pi} 22$

$\frac{\log 22}{\log \pi} \approx$

30) 2.7002

Solve the equation by expressing each side as a power of the same base and then equating exponents.

31) $4^{(x - 8)/2} = \sqrt{4}$

$4^{\frac{x-8}{2}} = 4^{\frac{1}{2}}$ $x=9$

31) $x=9$

32) $9^{x + 9} = 27^{x - 3}$

$(3^2)^{x+9} = (3^3)^{x-3}$ $2x+18=3x-9$

32) $x=27$

33) $25^x = \frac{1}{\sqrt{5}}$

$5^{2x} = 5^{-\frac{1}{2}}$

33) $x=-\frac{1}{4}$

Solve the exponential equation. Express the solution set in terms of natural logarithms.

34) $4^{x + 4} = 5^{2x + 5}$

$\ln 4^{x+4} = \ln 5^{2x+5}$

$(x+4)\ln 4 = (2x+5)\ln 5$

$x\ln 4 + 4\ln 4 = 2x\ln 5 + 5\ln 5$

$x(\ln 4 - 2\ln 5) = 5\ln 5 - 4\ln 4$

$x = \frac{5\ln 5 - 4\ln 4}{\ln 4 - 2\ln 5}$

34) $x = \frac{5\ln 5 - 4\ln 4}{\ln 4 - 2\ln 5}$

35) $2^{x + 8} = 6$

$\ln 2^{x+8} = \ln 6$

$(x+8)\ln 2 = \ln 6$

35) $x = \frac{\ln 6}{\ln 2} - 8$

Solve the exponential equation. Use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

36) $4^{6x} = 3.3$

$\log_4 3.3 = 6x$ $\frac{\log 3.3}{6 \cdot \log 4} = x \approx$

36) 0.14

37) $e^{5x - 7} - 3 = 1145$

$e^{5x-7} = 1148$

$\ln e^{5x-7} = \ln 1148$, $5x-7 = \ln 1148$

$x = \frac{\ln 1148 + 7}{5} \approx$

37) 2.81

Solve the logarithmic equation. Be sure to reject any value that is not in the domain of the original logarithmic expressions. Give the exact answer.

38) $\log_5 x + \log_5 (x - 24) = 2$

38) $x=25$

$\log_5 [x(x-24)] = 2$

$5^2 = x^2 - 24x$

$0 = x^2 - 24x - 25$

$0 = (x-25)(x+1)$

$x = 25, \cancel{-1}$

39) $\log_3 (x + 6) + \log_3 (x - 6) - \log_3 x = 2$ $\log_3 \frac{(x+6)(x-6)}{x} = 2$ $3^2 = \frac{x^2-36}{x}$

$9x = x^2 - 36;\ 0 = x^2 - 9x - 36$

$(x-12)(x+3) = 0$

$x = 12, \cancel{-3}$

39) $x = 12$

40) $\log_5 (x - 2) = 4 + \log_5 (x - 3)$

$\log_5 \left(\frac{x-2}{x-3}\right) = 4;\ 5^4 = \frac{x-2}{x-3};\ 625x - 1875 = x - 2$

$624x = 1873$

$x = \frac{1873}{624}$

40) $x = \frac{1873}{624}$

41) $\log_3 x^2 = \log_3 (5x + 36)$

$x^2 = 5x + 36;\ x^2 - 5x - 36 = 0;\ (x-9)(x+4) = 0$

$x = 9, -4$

41) $x = 9, -4$

42) $\ln (x - 6) + \ln (x + 1) = \ln (x - 15)$

$\ln((x-6)(x+1)) = \ln(x-15);\ x^2 - 5x - 6 = x - 15$

$x^2 - 6x + 9 = 0;\ (x-3)^2 = 0$

$x = \cancel{3}$

42) no Solution

43) $\ln (x - 2) - \ln (x + 4) = \ln (x - 5) - \ln (x + 2)$

$\ln\left(\frac{x-2}{x+4}\right) = \ln\left(\frac{x-5}{x+2}\right)$ $x^2 - 4 = x^2 - x - 20$

$x = -16$

43) no Solution

Solve the problem.

44) Find out how long it takes a \$2800 investment to double if it is invested at 8% compounded quarterly. Round to the nearest tenth of a year. Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

$2 = \left(1 + \frac{.08}{4}\right)^{4t};\ \log 2 = \log\left(1 + \frac{.08}{4}\right)^{4t};\ \log 2 = 4t \log\left(1 + \frac{.08}{4}\right);\ t = \frac{\log 2}{4 \log\left(1 + \frac{.08}{4}\right)}$

44) $t = 8.8$ yrs

45) The formula $A = 239e^{0.04t}$ models the population of a particular city, in thousands, t years after 1998. When will the population of the city reach 402 thousand?

$402 = 239e^{0.04t};\ \ln\left(\frac{402}{239}\right) = 0.04t;\ t = \ln\left(\frac{402}{239}\right) \div 0.04;\ t \approx 12.993$

45) $t = 2011$ +1998

46) Cindy will require \$10,000 in 2 years to return to college to get an MBA degree. How much money should she ask her parents for now so that, if she invests it at 12% compounded continuously, she will have enough for school? (Round your answer to the nearest dollar.)

$A = Pe^{rt};\ 10000 = Pe^{.12(2)};\ P = \frac{10000}{e^{.12(2)}}$

46) $P = \$7866.28$

47) The size of the coyote population at a national park increases at the rate of 4.8% per year. If the size of the current population is 200, find how many coyotes there should be in 6 years. Use $y = y_0e^{0.048t}$ and round to the nearest whole number.

$y = 200e^{.048(6)} =$

47) $y = 267$

Solve.

48) The function $A = A_0e^{-0.00866x}$ models the amount in pounds of a particular radioactive material stored in a concrete vault, where x is the number of years since the material was put into the vault. If 500 pounds of the material are initially put into the vault, how many pounds will be left after 170 years?

$A = 500e^{-.00866(170)}$

48) $A = 114.7$ lbs.

49) The half-life of silicon-32 is 710 years. If 20 grams is present now, how much will be present in 300 years? (Round your answer to three decimal places.)

$A = A_0e^{kt}$ $\frac{1}{2} = e^{k(710)};\ \frac{\ln(\frac{1}{2})}{710} = k;\ A = 20e^{\left(\frac{\ln(\frac{1}{2})}{710}\right)(300)}$

49) $A = 14.922$ g

50) A fossilized leaf contains 11% of its normal amount of carbon 14. How old is the fossil (to the nearest year)? Use 5600 years as the half-life of carbon 14.

$\frac{1}{2} = e^{k(5600)};\ \frac{\ln(\frac{1}{2})}{5600} = k;\ .11 = e^{\left(\frac{\ln(\frac{1}{2})}{5600}\right)t}$

$\frac{\ln .11}{\frac{\ln(\frac{1}{2})}{5600}} = t$

50) $t = 17833$ yrs