

Name _____

Key

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Complete the identity.

$$1) \sec x - \frac{1}{\sec x} = ? \quad \frac{\sec^2 x - 1}{\sec x} \rightarrow \frac{\tan^2 x}{\sec x} \rightarrow \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos x}{1} = \frac{\sin x \sin x}{\cos x} = \tan x \sin x \quad 1) \underline{\hspace{2cm}} \text{A}$$

- (A) $\sin x \tan x$ (B) $-2 \tan^2 x$ (C) $1 + \cot x$ (D) $\sec x \csc x$

$$2) \frac{(\sin x + \cos x)^2}{1 + 2 \sin x \cos x} = ? \quad \frac{(\sin x + \cos x)(\sin x + \cos x)}{1 + 2 \sin x \cos x} = \frac{\sin^2 x + 2 \sin x \cos x + \cos^2 x}{1 + 2 \sin x \cos x} = 1 \quad 2) \underline{\hspace{2cm}} \text{B}$$

- (A) $1 - \sin x$ (B) 1 (C) 0 (D) $-\sec^2 x$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Verify the identity.

$$3) \cot \theta \cdot \sec \theta = \csc \theta \quad \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} = \csc \theta \quad 3) \underline{\hspace{2cm}} \leftarrow$$

$$4) (1 + \tan^2 u)(1 - \sin^2 u) = 1 \quad \frac{\sec^2 u \cdot \cos^2 u}{\cos^2 u \cdot \cos^2 u} = 1 \quad 4) \underline{\hspace{2cm}} \leftarrow$$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the exact value of the expression.

$$5) \cos(260^\circ - 20^\circ) \quad \cos 240^\circ \quad \begin{array}{c} 180^\circ \\ 60^\circ \end{array} \quad \sqrt{3}/2 \quad 5) \underline{\hspace{2cm}} \text{A}$$

$$6) \cos\left(\frac{5\pi}{12} - \frac{\pi}{4}\right) \quad \cos\left(\frac{5\pi}{12} - \frac{3\pi}{12}\right) = \cos\left(\frac{2\pi}{12}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \quad 6) \underline{\hspace{2cm}} \text{A}$$

- (A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{1}{4}$

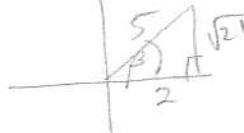
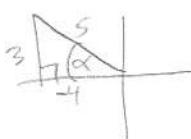
$$7) \cos(155^\circ) \cos(35^\circ) + \sin(155^\circ) \sin(35^\circ) \quad \cos(155^\circ - 35^\circ) = \cos(120^\circ) \quad \begin{array}{c} \sqrt{3}/2 \\ -1 \end{array} \quad 7) \underline{\hspace{2cm}} \text{C}$$

- (A) $-\sqrt{3}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $-\frac{1}{2}$ (D) -2

Use the given information to find the exact value of the expression.

$$8) \sin \alpha = \frac{3}{5}, \alpha \text{ lies in quadrant II, and } \cos \beta = \frac{2}{5}, \beta \text{ lies in quadrant I} \quad \text{Find } \cos(\alpha - \beta). \quad 8) \underline{\hspace{2cm}} \text{C}$$

$$\begin{array}{ll} \text{A) } \frac{6 - 4\sqrt{21}}{25} & \text{B) } \frac{6 + 4\sqrt{21}}{25} \\ \text{C) } \frac{-8 + 3\sqrt{21}}{25} & \text{D) } \frac{8 - 3\sqrt{21}}{25} \end{array}$$



$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\begin{aligned} &= -\frac{4}{5} \left(\frac{2}{5}\right) + \frac{3}{5} \left(\frac{\sqrt{21}}{5}\right) \\ &= -\frac{8}{25} + \frac{3\sqrt{21}}{25} \end{aligned}$$

51 A

Find the exact value by using a sum or difference identity.

$$9) \sin 165^\circ = \sin(45 + 120) = \sin 45 \cos 120 + \cos 45 \sin 120 = \frac{\sqrt{2}}{2} \left(\frac{-1}{2}\right) + \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2}\right) = \frac{-\sqrt{2} + \sqrt{2}\sqrt{3}}{4} = \frac{\sqrt{2}(-1 + \sqrt{3})}{4}$$

9) D

$$10) \cos(45^\circ + 60^\circ) = \cos 45 \cos 60 - \sin 45 \sin 60 = \frac{\sqrt{2}}{2} \left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2} - \sqrt{2}\sqrt{3}}{4} = \frac{\sqrt{2}(1 - \sqrt{3})}{4}$$

10) B

Find the exact value of the expression.

$$11) \sin 185^\circ \cos 65^\circ - \cos 185^\circ \sin 65^\circ = \sin(185 - 65) = \sin 120$$

11) D

- A) $-\frac{\sqrt{3}}{2}$
 B) $-\frac{1}{2}$
 C) $\frac{37}{12}$
 D) $\frac{\sqrt{3}}{2}$

$$12) \cos \frac{2\pi}{9} \sin \frac{\pi}{18} \cos \frac{\pi}{18} \sin \frac{2\pi}{9} = \sin \left(\frac{\pi}{18} - \frac{2\pi}{9}\right) = \sin \left(\frac{\pi}{18} - \frac{4\pi}{18}\right) = \sin \left(-\frac{3\pi}{18}\right) = \sin \left(-\frac{\pi}{6}\right)$$

12) -\frac{1}{2}

- A) $\frac{\sqrt{3}}{2}$
 B) $\frac{1}{4}$
 C) $\frac{1}{2}$
 D) 1

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Verify the identity.

$$13) \cos \left(x + \frac{\pi}{2}\right) = -\sin x$$

13) ←

$$14) \sin \left(\frac{3\pi}{2} - \theta\right) = -\cos \theta$$

14) ←

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the given information to find the exact value of the expression.

$$15) \sin \alpha = \frac{21}{29}, \alpha \text{ lies in quadrant I, and } \cos \beta = \frac{5}{13}, \beta \text{ lies in quadrant I}$$

Find $\cos(\alpha + \beta)$.

A) $-\frac{135}{377}$ B) $-\frac{152}{377}$ C) $\frac{345}{377}$ D) $\frac{352}{377}$

15) B

-

$$16) \cos \alpha = -\frac{24}{25}, \alpha \text{ lies in quadrant III, and } \sin \beta = \frac{\sqrt{21}}{5}, \beta \text{ lies in quadrant II}$$

Find $\cos(\alpha + \beta)$.

A) $-\frac{14 + 24\sqrt{21}}{125}$ B) $\frac{48 + 7\sqrt{21}}{125}$ C) $-\frac{48 - 7\sqrt{21}}{125}$ D) $\frac{14 - 24\sqrt{21}}{125}$

16) B

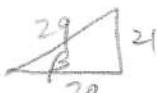
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Use trigonometric identities to find the exact value.

$$17) \frac{\tan 40^\circ + \tan 110^\circ}{1 - \tan 40^\circ \tan 110^\circ} = \tan 150$$

17) C

- A) -2
 B) $-\sqrt{3}$
 C) $-\frac{\sqrt{3}}{3}$
 D) $-\frac{1}{2}$



Find the exact value under the given conditions.

$$18) \sin \alpha = \frac{24}{25}, 0 < \alpha < \frac{\pi}{2}; \cos \beta = \frac{20}{29}, 0 < \beta < \frac{\pi}{2} \quad \text{Find } \tan(\alpha + \beta).$$

$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{24}{7} + \frac{21}{20}}{1 - \frac{24}{7} \cdot \frac{21}{20}} = \frac{627}{364}$$

B

A) $\frac{627}{725}$

B) $-\frac{627}{364}$

C) $-\frac{364}{725}$

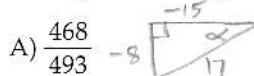
D) $\frac{644}{725}$

$$19) \tan \alpha = \frac{15}{8}, \pi < \alpha < \frac{3\pi}{2}; \cos \beta = -\frac{21}{29}, \frac{\pi}{2} < \beta < \pi \quad \text{Find } \tan(\alpha + \beta).$$

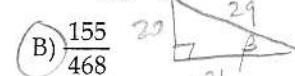
$$\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{15}{8} + \frac{20}{-21}}{1 - \frac{15}{8} \cdot \frac{20}{-21}}$$

B

A) $\frac{468}{493}$



B) $\frac{155}{468}$



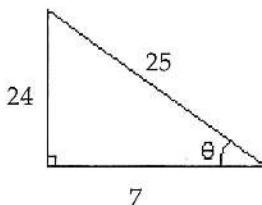
C) $-\frac{11}{39}$

D) $\frac{155}{493}$

Use the figure to find the exact value of the trigonometric function.

20) Find $\tan 2\theta$.

20) D



$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \left(\frac{24}{7} \right)}{1 - \left(\frac{24}{7} \right)^2} = -\frac{336}{527}$$

A) $-\frac{527}{625}$

B) $\frac{336}{625}$

C) $\frac{528}{527}$

D) $-\frac{336}{527}$

Use the given information to find the exact value of the expression.

$$21) \cos \theta = \frac{21}{29}, \theta \text{ lies in quadrant IV.} \quad \text{Find } \sin 2\theta.$$

$$\begin{aligned} \sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left(-\frac{20}{29} \right) \left(\frac{21}{29} \right) \end{aligned}$$

A

A) $-\frac{840}{841}$

B) $\frac{840}{841}$

C) $-\frac{41}{841}$

D) $\frac{41}{841} = -\frac{840}{841}$

$$22) \sin \theta = \frac{24}{25}, \theta \text{ lies in quadrant II.} \quad \text{Find } \tan 2\theta.$$

$$\begin{aligned} \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \left(\frac{24}{7} \right)}{1 - \left(\frac{24}{7} \right)^2} \\ &= \frac{336}{527} \end{aligned}$$

A) $-\frac{336}{527}$

B) $\frac{336}{527}$

C) $-\frac{526}{527}$

D) $\frac{336}{625} = +\frac{336}{527}$

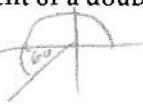
Write the expression as the sine, cosine, or tangent of a double angle. Then find the exact value of the expression.

23) $2 \sin 120^\circ \cos 120^\circ$

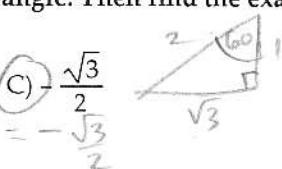
23) C

A) $\frac{1}{2}$

$\sin 2(120^\circ) = \sin 240^\circ$



C) $-\frac{\sqrt{3}}{2}$



D) $\frac{\sqrt{3}}{2}$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Verify the identity.

24) $\cos 4\theta = 2 \cos^2(2\theta) - 1$

24) ←

$\cos 2(2\theta)$
 $2 \cos^2(2\theta) - 1$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Rewrite the expression as an equivalent expression that does not contain powers of trigonometric functions greater than 1.

25) $8 \cos^2 x = 8 \left(\frac{1 + \cos 2x}{2} \right) = 4(1 + \cos 2x) = 4 + 4\cos 2x$

25) C

- A) $1 + \cos 2x$ B) $4 - 4 \cos 2x$

- C) $4 + 4 \cos 2x$

- D) $16 \cos x$

26) $2 \sin^2 x \cos^2 x = 2 \left(\frac{1 - \cos 2x}{2} \right) \left(\frac{1 + \cos 2x}{2} \right) = \frac{1 - \cos^2 2x}{2} = \frac{1 - (1 + \cos 2(2x))}{2}$

26) D

- A) $\frac{1}{2} - \frac{1}{2} \cos 4x$ B) $\frac{1}{2} \sin 2x$

- C) $\frac{1}{4} - \frac{1}{4} \cos 2x$

- D) $\frac{1}{4} - \frac{1}{4} \cos 4x$

Use a half-angle formula to find the exact value of the expression.

27) $\sin 75^\circ$

27) A

(A) $\frac{1}{2} \sqrt{2 + \sqrt{3}}$

$\sin 75^\circ = \sqrt{\frac{1 - \cos 150}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{4}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$

B) $\frac{1}{2} \sqrt{2 - \sqrt{3}}$

$\sin 75^\circ = \sqrt{\frac{1 - \cos 150}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$

C) $-\frac{1}{2} \sqrt{2 - \sqrt{3}}$

D) $-\frac{1}{2} \sqrt{2 + \sqrt{3}}$

28) $\cos \frac{3\pi}{8}$

28) A

(A) $\frac{1}{2} \sqrt{2 - \sqrt{2}}$

$\cos \frac{3\pi}{8} = \sqrt{\frac{1 + \cos \frac{3\pi}{4}}{2}} = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

B) $-\frac{1}{2} \sqrt{2 - \sqrt{2}}$

$\cos \frac{3\pi}{8} = \sqrt{\frac{1 + \cos \frac{3\pi}{4}}{2}} = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

C) $\frac{1}{2} \sqrt{2 + \sqrt{2}}$

D) $-\frac{1}{2} \sqrt{2 + \sqrt{2}}$

Use the given information to find the exact value of the trigonometric function.

29) $\sin \theta = \frac{1}{4}$, θ lies in quadrant I

Find $\sin \frac{\theta}{2}$.

(A) $\frac{\sqrt{8 - 2\sqrt{15}}}{4}$

$\sin \theta = \frac{1}{4}$

(B) $\frac{\sqrt{8 + 2\sqrt{15}}}{4}$

$\sin \theta = \frac{1}{4}$

C) $\frac{\sqrt{6}}{4}$

D) $\frac{\sqrt{10}}{4}$

$\frac{\sqrt{4 - \sqrt{15}} \cdot \sqrt{2}}{2\sqrt{2}} = \frac{\sqrt{8 - 2\sqrt{15}}}{4}$

29) A

30) $\sec \theta = 4$, θ lies in quadrant I

Find $\cos \frac{\theta}{2}$.

(A) $\frac{\sqrt{8 - 2\sqrt{15}}}{4}$

$\sec \theta = 4$

(B) $\frac{\sqrt{10}}{4}$

$\sec \theta = 4$

C) $\frac{\sqrt{6}}{4}$

D) $\frac{\sqrt{8 + 2\sqrt{15}}}{4}$

$\frac{\sqrt{4 + \sqrt{15}} \cdot \sqrt{2}}{2\sqrt{2}} = \frac{\sqrt{8 + 2\sqrt{15}}}{4}$

30) B

31) $\sin \theta = -\frac{3}{5}$, θ lies in quadrant IV

Find $\sin \frac{\theta}{2}$.

(A) $-\frac{\sqrt{5}}{5}$

$\sin \theta = -\frac{3}{5}$

(B) $-\frac{\sqrt{30}}{10}$

$\sin \theta = -\frac{3}{5}$

(C) $\frac{\sqrt{10}}{10}$

D) $\frac{\sqrt{5}}{5}$

$\frac{\sqrt{1 - \cos \theta}}{2} = \sqrt{1 - \frac{-4}{5}} = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}} = \frac{\sqrt{15}}{5}$

31) C

Express the product as a sum or difference.

32) $\sin 8x \cos 4x = \frac{1}{2} [\sin(8x+4x) + \sin(8x-4x)]$

32) C

A) $\frac{1}{2}(\sin 12x + \cos 4x)$

B) $\frac{1}{2}(\cos 12x - \cos 4x)$

C) $\frac{1}{2}(\sin 12x + \sin 4x)$

D) $\sin(\cos 32x^2)$

$= \frac{1}{2} [\sin 12x + \sin 4x]$

33) $\sin 2x \sin 5x$

$$\begin{aligned} & \frac{1}{2} [\cos(2x-5x) - \cos(2x+5x)] \\ & A) \frac{1}{2}(\cos 7x - \sin 3x) \\ & = \frac{1}{2} [\cos(-3x) \overset{\text{even}}{-} \cos 7x] \quad B) \frac{1}{2}(-\cos 3x - \cos 7x) \\ & C) \frac{1}{2}(\cos 3x - \cos 7x) \quad D) \sin^2 10x^2 \\ & \frac{1}{2} (-\cos 3x - \cos 7x) \end{aligned}$$

33) C

Express the sum or difference as a product.

34) $\cos 9x - \cos 3x$

$$= -2 \sin \frac{9x+3x}{2} \sin \frac{9x-3x}{2} = -2 \sin 6x \sin 3x$$

A) $-2 \cos 6x \sin 3x$ B) $-2 \sin 6x \sin 3x$ C) $2 \cos 3x$ D) $2 \cos 6x \cos 3x$

34) B

35) $\sin 8x - \sin 4x$

$$= 2 \sin \frac{8x-4x}{2} \cos \frac{8x+4x}{2} = 2 \sin 2x \cos 6x$$

A) $2 \cos 4x \cos 6x$ B) $2 \sin 6x \cos 2x$ C) $2 \sin 2x \cos 6x$ D) $2 \sin 2x$

35) C

Use substitution to determine whether the given x-value is a solution of the equation.

36) $\sin x = -\frac{\sqrt{3}}{2}, x = -\frac{2\pi}{3}$

$$\sin -\frac{2\pi}{3} \overset{?}{=} -\frac{\sqrt{3}}{2} \quad \begin{array}{|c|} \hline 2 & 60^\circ \\ \hline 30 & \\ \hline \end{array} \quad 1.$$

A) Yes B) No $\sqrt{3}$

36) A

37) $\tan x = \frac{\sqrt{3}}{3}, x = \frac{7\pi}{6}$

$$\tan \frac{7\pi}{6} \overset{?}{=} \frac{\sqrt{3}}{3} \quad \begin{array}{|c|} \hline 2 & 30^\circ \\ \hline 30 & \\ \hline \end{array} \quad 1.$$

A) Yes B) No

37) A

Find all solutions of the equation.

38) $\tan x = \frac{\sqrt{3}}{3}$

$$\begin{array}{|c|} \hline 2 & 30^\circ \\ \hline 30 & \\ \hline \end{array} \quad x = \frac{\pi}{6} + n\pi \quad \text{period}$$

A) $x = \frac{5\pi}{6} + 2n\pi$ B) $x = \frac{5\pi}{6} + n\pi$ C) $x = \frac{\pi}{6} + n\pi$ D) $x = \frac{2\pi}{3} + 2n\pi$

38) C

39) $2 \sin x - \sqrt{3} = 0$

$$\begin{aligned} \sin x &= \frac{\sqrt{3}}{2} \quad \begin{array}{|c|} \hline 2 & 60^\circ \\ \hline 30 & \\ \hline \end{array} \quad x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ X &= \frac{\pi}{3} + 2n\pi \quad B) x = \frac{\pi}{6} + 2n\pi \text{ or } x = \frac{5\pi}{3} + 2n\pi \\ &\text{or} \\ X &= \frac{2\pi}{3} + 2n\pi \quad D) x = \frac{\pi}{3} + 2n\pi \text{ or } x = \frac{2\pi}{3} + 2n\pi \end{aligned}$$

39) D

40) $5 \sin x - 8\sqrt{2} = 3 \sin x - 7\sqrt{2}$

$$5 \sin x - 3 \sin x - 7\sqrt{2} = 0$$

A) $x = \frac{\pi}{4} + 2n\pi$ or $x = \frac{3\pi}{4} + 2n\pi$ B) $x = \frac{5\pi}{4} + n\pi$ or $x = \frac{7\pi}{4} + n\pi$
C) $x = \frac{5\pi}{4} + 2n\pi$ or $x = \frac{7\pi}{4} + 2n\pi$ D) $x = \frac{\pi}{4} + n\pi$ or $x = \frac{3\pi}{4} + n\pi$

40) A

Solve the equation on the interval $[0, 2\pi]$.

41) $\cos 2x = \frac{\sqrt{2}}{2}$

$$2x = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

A) $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ B) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

41) C

C) $\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}$

$$\begin{aligned} n=0: & \frac{\pi}{8}, \frac{7\pi}{8} \\ n=1: & \frac{9\pi}{8}, \frac{15\pi}{8} \end{aligned}$$

42) $\cos^2 x + 2 \cos x + 1 = 0$

(A) π (B) 2π

$$(\cos x + 1)(\cos x + 1) = 0$$

$$\cos x = -1$$

$$x = \cos^{-1}(-1)$$

$$x = \pi$$

C) $\frac{\pi}{4}, \frac{7\pi}{4}$

D) $\frac{\pi}{2}, \frac{3\pi}{2}$

42)

A

43) $2 \sin^2 x = \sin x$

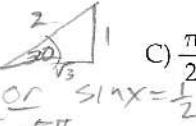
A) $\frac{\pi}{3}, \frac{2\pi}{3}$

$2 \sin^2 x - \sin x = 0$

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B) $\frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x = 0$ or $\sin x = \frac{1}{2}$



$\sin x = \frac{1}{2}$

C) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$

D) $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

43)

D

44) $\cos x = \sin x$

A) $\frac{\pi}{4}, \frac{7\pi}{4}$

$\cos^2 x = \sin^2 x$

Square both sides: check solns

B) $\frac{3\pi}{4}, \frac{5\pi}{4}$

$\cos 2x = 0$

$2x = \frac{\pi}{2} + k\pi$ or $2x = \frac{3\pi}{2} + k\pi$

$x = \frac{\pi}{4} + \frac{k\pi}{2}$ or $x = \frac{3\pi}{4} + \frac{k\pi}{2}$

C) $\frac{3\pi}{4}, \frac{7\pi}{4}$

$n = 0 = \frac{\pi}{4}$

$n = 1 = \frac{5\pi}{4}$

$n = 2 = \frac{9\pi}{4}$

$n = 3 = \frac{13\pi}{4}$

$n = 4 = \frac{17\pi}{4}$

$n = 5 = \frac{21\pi}{4}$

$n = 6 = \frac{25\pi}{4}$

$n = 7 = \frac{29\pi}{4}$

$n = 8 = \frac{33\pi}{4}$

$n = 9 = \frac{37\pi}{4}$

$n = 10 = \frac{41\pi}{4}$

$n = 11 = \frac{45\pi}{4}$

$n = 12 = \frac{49\pi}{4}$

$n = 13 = \frac{53\pi}{4}$

$n = 14 = \frac{57\pi}{4}$

$n = 15 = \frac{61\pi}{4}$

$n = 16 = \frac{65\pi}{4}$

$n = 17 = \frac{69\pi}{4}$

$n = 18 = \frac{73\pi}{4}$

$n = 19 = \frac{77\pi}{4}$

$n = 20 = \frac{81\pi}{4}$

$n = 21 = \frac{85\pi}{4}$

$n = 22 = \frac{89\pi}{4}$

$n = 23 = \frac{93\pi}{4}$

$n = 24 = \frac{97\pi}{4}$

$n = 25 = \frac{101\pi}{4}$

$n = 26 = \frac{105\pi}{4}$

$n = 27 = \frac{109\pi}{4}$

$n = 28 = \frac{113\pi}{4}$

$n = 29 = \frac{117\pi}{4}$

$n = 30 = \frac{121\pi}{4}$

$n = 31 = \frac{125\pi}{4}$

$n = 32 = \frac{129\pi}{4}$

$n = 33 = \frac{133\pi}{4}$

$n = 34 = \frac{137\pi}{4}$

$n = 35 = \frac{141\pi}{4}$

$n = 36 = \frac{145\pi}{4}$

$n = 37 = \frac{149\pi}{4}$

$n = 38 = \frac{153\pi}{4}$

$n = 39 = \frac{157\pi}{4}$

$n = 40 = \frac{161\pi}{4}$

$n = 41 = \frac{165\pi}{4}$

$n = 42 = \frac{169\pi}{4}$

$n = 43 = \frac{173\pi}{4}$

$n = 44 = \frac{177\pi}{4}$

$n = 45 = \frac{181\pi}{4}$

$n = 46 = \frac{185\pi}{4}$

$n = 47 = \frac{189\pi}{4}$

$n = 48 = \frac{193\pi}{4}$

$n = 49 = \frac{197\pi}{4}$

$n = 50 = \frac{201\pi}{4}$

$n = 51 = \frac{205\pi}{4}$

$n = 52 = \frac{209\pi}{4}$

$n = 53 = \frac{213\pi}{4}$

$n = 54 = \frac{217\pi}{4}$

$n = 55 = \frac{221\pi}{4}$

$n = 56 = \frac{225\pi}{4}$

$n = 57 = \frac{229\pi}{4}$

$n = 58 = \frac{233\pi}{4}$

$n = 59 = \frac{237\pi}{4}$

$n = 60 = \frac{241\pi}{4}$

$n = 61 = \frac{245\pi}{4}$

$n = 62 = \frac{249\pi}{4}$

$n = 63 = \frac{253\pi}{4}$

$n = 64 = \frac{257\pi}{4}$

$n = 65 = \frac{261\pi}{4}$

$n = 66 = \frac{265\pi}{4}$

$n = 67 = \frac{269\pi}{4}$

$n = 68 = \frac{273\pi}{4}$

$n = 69 = \frac{277\pi}{4}$

$n = 70 = \frac{281\pi}{4}$

$n = 71 = \frac{285\pi}{4}$

$n = 72 = \frac{289\pi}{4}$

$n = 73 = \frac{293\pi}{4}$

$n = 74 = \frac{297\pi}{4}$

$n = 75 = \frac{301\pi}{4}$

$n = 76 = \frac{305\pi}{4}$

$n = 77 = \frac{309\pi}{4}$

$n = 78 = \frac{313\pi}{4}$

$n = 79 = \frac{317\pi}{4}$

$n = 80 = \frac{321\pi}{4}$

$n = 81 = \frac{325\pi}{4}$

$n = 82 = \frac{329\pi}{4}$

$n = 83 = \frac{333\pi}{4}$

$n = 84 = \frac{337\pi}{4}$

$n = 85 = \frac{341\pi}{4}$

$n = 86 = \frac{345\pi}{4}$

$n = 87 = \frac{349\pi}{4}$

$n = 88 = \frac{353\pi}{4}$

$n = 89 = \frac{357\pi}{4}$

$n = 90 = \frac{361\pi}{4}$

$n = 91 = \frac{365\pi}{4}$

$n = 92 = \frac{369\pi}{4}$

$n = 93 = \frac{373\pi}{4}$

$n = 94 = \frac{377\pi}{4}$

$n = 95 = \frac{381\pi}{4}$

$n = 96 = \frac{385\pi}{4}$

$n = 97 = \frac{389\pi}{4}$

$n = 98 = \frac{393\pi}{4}$

$n = 99 = \frac{397\pi}{4}$

$n = 100 = \frac{401\pi}{4}$

$n = 101 = \frac{405\pi}{4}$

$n = 102 = \frac{409\pi}{4}$

$n = 103 = \frac{413\pi}{4}$

$n = 104 = \frac{417\pi}{4}$

$n = 105 = \frac{421\pi}{4}$

$n = 106 = \frac{425\pi}{4}$

$n = 107 = \frac{429\pi}{4}$

$n = 108 = \frac{433\pi}{4}$

$n = 109 = \frac{437\pi}{4}$

$n = 110 = \frac{441\pi}{4}$

$n = 111 = \frac{445\pi}{4}$

$n = 112 = \frac{449\pi}{4}$

$n = 113 = \frac{453\pi}{4}$

$n = 114 = \frac{457\pi}{4}$

$n = 115 = \frac{461\pi}{4}$

$n = 116 = \frac{465\pi}{4}$

$n = 117 = \frac{469\pi}{4}$

$n = 118 = \frac{473\pi}{4}$

$n = 119 = \frac{477\pi}{4}$

$n = 120 = \frac{481\pi}{4}$

$n = 121 = \frac{485\pi}{4}$

$n = 122 = \frac{489\pi}{4}$

$n = 123 = \frac{493\pi}{4}$

$n = 124 = \frac{497\pi}{4}$

$n = 125 = \frac{501\pi}{4}$

$n = 126 = \frac{505\pi}{4}$

$n = 127 = \frac{509\pi}{4}$

$n = 128 = \frac{513\pi}{4}$

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$n = 131 = \frac{525\pi}{4}$

$n = 132 = \frac{529\pi}{4}$

$n = 133 = \frac{533\pi}{4}$

$n = 134 = \frac{537\pi}{4}$

$n = 135 = \frac{541\pi}{4}$

$n = 136 = \frac{545\pi}{4}$

$n = 137 = \frac{549\pi}{4}$

$n = 138 = \frac{553\pi}{4}$

$n = 139 = \frac{557\pi}{4}$

$n = 140 = \frac{561\pi}{4}$

$n = 141 = \frac{565\pi}{4}$

$n = 142 = \frac{569\pi}{4}$

$n = 143 = \frac{573\pi}{4}$

$n = 144 = \frac{577\pi}{4}$

$n = 145 = \frac{581\pi}{4}$

$n = 146 = \frac{585\pi}{4}$

$n = 147 = \frac{589\pi}{4}$

$n = 148 = \frac{593\pi}{4}$

$n = 149 = \frac{597\pi}{4}$

$n = 150 = \frac{601\pi}{4}$

$n = 151 = \frac{605\pi}{4}$

$n = 152 = \frac{609\pi}{4}$

$n = 153 = \frac{613\pi}{4}$

$n = 154 = \frac{617\pi}{4}$

$n = 155 = \frac{621\pi}{4}$

$n = 156 = \frac{625\pi}{4}$

$n = 157 = \frac{629\pi}{4}$

$n = 158 = \frac{633\pi}{4}$

$n = 159 = \frac{637\pi}{4}$

$n = 160 = \frac{641\pi}{4}$

$n = 161 = \frac{645\pi}{4}$

$n = 162 = \frac{649\pi}{4}$

$n = 163 = \frac{653\pi}{4}$

$n = 164 = \frac{657\pi}{4}$

$n = 165 = \frac{661\pi}{4}$

$n = 166 = \frac{665\pi}{4}$

$n = 167 = \frac{669\pi}{4}$

$n = 168 = \frac{673\$