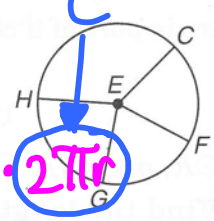
NAME _____ DATE _____ PERIOD ____

10-2 Study Guide and Intervention

Measuring Angles and Arcs

11.5 Arc Lengths (supplement) → length of the "crust"

Angles and Arcs A **central angle** is an angle whose vertex is at the center of a circle and whose sides are radii. A central angle separates a circle into two arcs, a **major arc** and a **minor arc**.



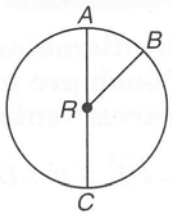
$\overset{\frown}{GF}$ is a minor arc.
$\overset{\frown}{CHG}$ is a major arc.
$\angle GEF$ is a central angle.

Arc Length: $\ell = \frac{\text{degrees of arc}}{360} \cdot 2\pi r$

Here are some properties of central angles and arcs.

- The sum of the measures of the central angles of a circle with no interior points in common is 360. — $m\angle HEC + m\angle CEF + m\angle FEG + m\angle GEH = 360$
- The measure of a minor arc equals the measure of its central angle. — $m\overset{\frown}{CF} = m\angle CEF$
- The measure of a major arc is 360 minus the measure of the minor arc. — $m\overset{\frown}{CGF} = 360 - m\overset{\frown}{CF}$
- Two arcs are congruent if and only if their corresponding central angles are congruent. — $\overset{\frown}{CF} \cong \overset{\frown}{FG}$ if and only if $\angle CEF \cong \angle FEG$.
- The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. **(Arc Addition Postulate)** — $m\overset{\frown}{CF} + m\overset{\frown}{FG} = m\overset{\frown}{CG}$

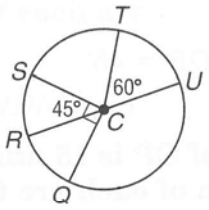
Example **In $\odot R$, $m\angle ARB = 42$ and $\overline{AC}$ is a diameter. Find $m\overset{\frown}{AB}$ and $m\overset{\frown}{ACB}$.**



$\angle ARB$ is a central angle and $m\angle ARB = 42$, so $m\overset{\frown}{AB} = 42$.
Thus $m\overset{\frown}{ACB} = 360 - 42$ or 318.

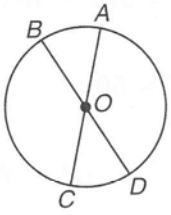
Exercises

Find each measure.



1. $m\angle SCT$
2. $m\angle SCU$
3. $m\angle SCQ$
4. $m\angle QCT$

In $\odot O$, $m\angle BOA = 44$. Find each measure.



5. $m\overset{\frown}{BA}$
6. $m\overset{\frown}{BC}$
7. $m\overset{\frown}{CD}$
8. $m\overset{\frown}{ACB}$
9. $m\overset{\frown}{BCD}$
10. $m\overset{\frown}{AD}$

Lesson 10-2

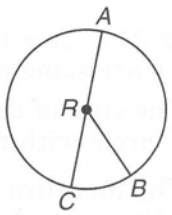
NAME _____ DATE _____ PERIOD _____

10-2 Study Guide and Intervention *(continued)*

Measuring Angles and Arcs

Arc Length An arc is part of a circle and its length is a part of the circumference of the circle.

Example **In $\odot R$, $m\angle ARB = 135$, $RB = 8$, and AC is a diameter. Find the length of $\overset{\frown}{AB}$.**



$m\angle ARB = 135$, so $m\overset{\frown}{AB} = 135$. Using the formula $C = 2\pi r$, the circumference is $2\pi(8)$ or 16π. To find the length of $\overset{\frown}{AB}$, write a proportion to compare each part to its whole.

$$\frac{\text{length of } \overset{\frown}{AB}}{\text{circumference}} = \frac{\text{degree measure of arc}}{\text{degree measure of circle}}$$ Proportion

$$\frac{\ell}{16\pi} = \frac{135}{360}$$ Substitution

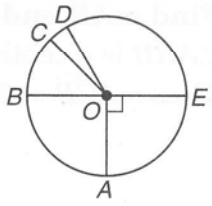
$$\ell = \frac{(16\pi)(135)}{360}$$ Multiply each side by 16π.

$$= 6\pi$$ Simplify.

The length of $\overset{\frown}{AB}$ is 6π or about 18.85 units.

Exercises

The diameter of $\odot O$ is 24 units long. Find the length of each arc for the given angle measure. Round to the nearest tenth.



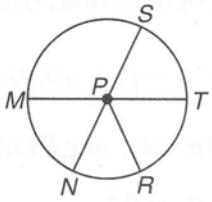
1. $\overset{\frown}{DE}$ if $m\angle DOE = 120$

2. $\overset{\frown}{DEA}$ if $m\angle DOE = 120$

3. $\overset{\frown}{BC}$ if $m\angle COB = 45$

4. $\overset{\frown}{CBA}$ if $m\angle COB = 45$

The diameter of $\odot P$ is 15 units long and $\angle SPT \cong \angle RPT$. Find the length of each arc for the given angle measure. Round to the nearest tenth.



5. $\overset{\frown}{RT}$ if $m\angle SPT = 70$

6. $\overset{\frown}{NR}$ if $m\angle RPT = 50$

7. $\overset{\frown}{MST}$

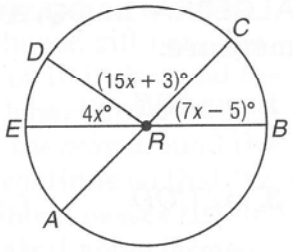
8. $\overset{\frown}{MRS}$ if $m\angle MPS = 140$

NAME _____ DATE _____ PERIOD _____

10-2 Skills Practice

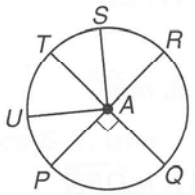
Measuring Angles and Arcs

ALGEBRA In $\odot R$, $\overline{AC}$ and $\overline{EB}$ are diameters. Find each measure.



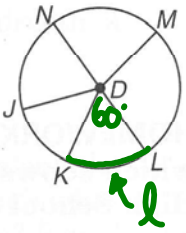
1. $m\angle ERD$
2. $m\angle CRD$
3. $m\angle BRC$
4. $m\angle ARB$
5. $m\angle ARE$
6. $m\angle BRD$

In $\odot A$, $m\angle PAU = 40$, $\angle PAU \cong \angle SAT$, and $\angle RAS \cong \angle TAU$. Find each measure.



7. $m\overset{\frown}{PQ}$
8. $m\overset{\frown}{PQR}$
9. $m\overset{\frown}{ST}$
10. $m\overset{\frown}{RS}$
11. $m\overset{\frown}{RSU}$
12. $m\overset{\frown}{STP}$
13. $m\overset{\frown}{PQS}$
14. $m\overset{\frown}{PRU}$

The diameter of $\odot D$ is 18 units long. Find the length of each arc for the given angle measure.



15. $\overset{\frown}{LM}$ if $m\angle LDM = 100$
16. $\overset{\frown}{MN}$ if $m\angle MDN = 80$
17. $\overset{\frown}{KL}$ if $m\angle KDL = 60$
18. $\overset{\frown}{NJK}$ if $m\angle NDK = 120$

$\ell = \frac{N}{360} \cdot 2\pi r = \frac{60}{360} \cdot 2\pi(9) = 9.4$ units

19. $\overset{\frown}{KLM}$ if $m\angle KDM = 160$
20. $\overset{\frown}{JK}$ if $m\angle JDK = 50$

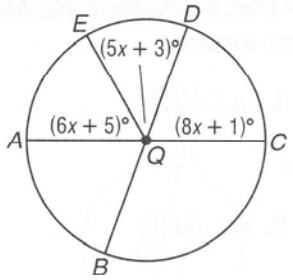
Lesson 10-2

NAME _____ DATE _____ PERIOD _____

10-2 Practice

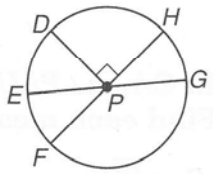
Measuring Angles and Arcs

ALGEBRA In $\odot Q$, $\overline{AC}$ and $\overline{BD}$ are diameters. Find each measure.



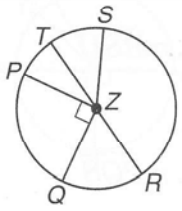
1. $m\angle AQE$
2. $m\angle DQE$
3. $m\angle CQD$
4. $m\angle BQC$
5. $m\angle CQE$
6. $m\angle AQD$

In $\odot P$, $m\angle GPH = 38$. Find each measure.



7. $m\widehat{EF}$
8. $m\widehat{DE}$
9. $m\widehat{FG}$
10. $m\widehat{DHG}$
11. $m\widehat{DFG}$
12. $m\widehat{DGE}$

The radius of $\odot Z$ is 13.5 units long. Find the length of each arc for the given angle measure.



13. $\widehat{QPT}$ if $m\angle QZT = 120$
14. $\widehat{QR}$ if $m\angle QZR = 60$
15. $\widehat{PQR}$ if $m\angle PZR = 150$
16. $\widehat{QPS}$ if $m\angle QZS = 160$

HOMEWORK For Exercises 17 and 18, refer to the table, which shows the number of hours students at Leland High School say they spend on homework each night.

Homework	
Less than 1 hour	8%
1–2 hours	29%
2–3 hours	58%
3–4 hours	3%
Over 4 hours	2%

17. If you were to construct a circle graph of the data, how many degrees would be allotted to each category?

18. Describe the arcs associated with each category.

10–2 Angles and Arcs

MAIN IDEAS

- Recognize major arcs, minor arcs, semicircles, and central angles and their measures.
- Find arc length.

KEY CONCEPT

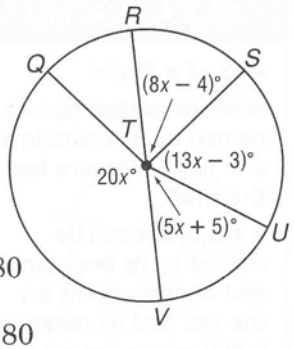
Sum of Central Angles
The sum of the measures of the central angles of a circle with no interior points in common is 360.

FOLDABLES Include this concept in your notes.

BUILD YOUR VOCABULARY (pages 244–245)

A **central angle** has the _____ of the circle as its vertex, and its sides contain two radii of the circle.

EXAMPLE Measures of Central Angles



1 ALGEBRA $\overline{RV}$ **is a diameter of** $\odot T$**. Find** $m\angle RTS$**.**

The sum of the measures of $\angle RTS$, $\angle STU$, and $\angle UTV$ is 180.

$$m\angle RTS + m\angle STU + m\angle UTV = 180$$

$(8x - 4) +$ _____ $+$ _____ $= 180$

_____ $= 180$ Simplify.

_____ $= 182$ Add 2.

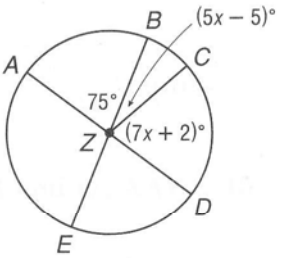
$x =$ _____ Divide.

Use the value of x to find $m\angle RTS$.

$m\angle RTS = 8x - 4$ Given

$= 8$ _____ $- 4$ or _____ Substitution

Check Your Progress Find $m\angle CZD$.



BUILD YOUR VOCABULARY (pages 244–245)

A central angle separates the circle into two parts, each of which is an **arc**.

Theorem 10.1
Two arcs are congruent if and only if their corresponding central angles are congruent.

Postulate 10.1 Arc Addition Postulate
The measure of an arc formed by two adjacent arcs is the sum of the measures of two arcs.

KEY CONCEPTS

Arcs of a Circle

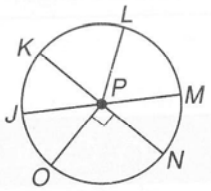
A **minor arc** can be named by its endpoints and has a measure less than 180.

A **major arc** can be named by its endpoints and another point on the arc, and its measure is 360 minus the measure of the related minor arc.

A **semicircle** can be named by its endpoints and another point on the arc, and its measure is 180.

EXAMPLE Measures of Arcs

2 In $\odot P$, $m\angle NPM = 46$, $\overline{PL}$ bisects $\angle KPM$, and $\overline{OP} \perp \overline{KN}$.



a. Find $m\widehat{OK}$.

$\widehat{OK}$ is a minor arc, so $m\widehat{OK} = m\angle KPO$.

$\widehat{KON}$ is a semicircle.

$m\widehat{ON} = m\angle NPO$

$= \square$ $\angle NPO$ is a right angle.

$m\widehat{KON} = m\widehat{OK} + m\widehat{ON}$ Arc Addition Postulate

$\square = m\widehat{OK} + \square$ Substitution

$\square = m\widehat{OK}$ Subtract.

b. Find $m\widehat{LM}$.

$m\widehat{LM} = \frac{1}{2}\widehat{KM}$ since $\overline{PL}$ bisects $\angle KPM$.

$\widehat{KMN}$ is a semicircle.

$m\widehat{KM} + m\widehat{MN} = m\widehat{KMN}$ Arc Addition Postulate

$m\widehat{KM} + \square = \square$ $m\widehat{MN} = m\angle NPM = 46$

$m\widehat{KM} = \square$ Subtract.

$m\widehat{LM} = \frac{1}{2}\square$ or 67

c. $m\widehat{JKO}$

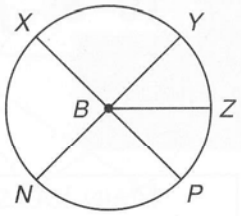
$\widehat{JKO}$ is a major arc.

$m\widehat{JKO} = m\widehat{JLM} + m\widehat{MN} + m\widehat{NO}$ Arc Addition Postulate

$m\widehat{JKO} = 180 + \square + \square$ Substitution

$m\widehat{JKO} = \square$

Check Your Progress **In $\odot B$, $\overline{XP}$ and and $\overline{YN}$ are diameters, $m\angle XBN = 108$, and $\overline{BZ}$ bisects $\angle YBP$. Find each measure.**



a. $m\widehat{YZ}$

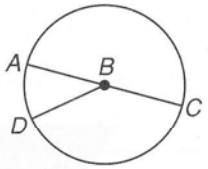
b. $m\widehat{XY}$

c. $m\widehat{XNZ}$

EXAMPLE **Arc Length**

3 **In $\odot B$, $AC = 9$ and $m\angle ABD = 40$. Find the length of $\widehat{AD}$.**

In $\odot B$, $AC = 9$ so $C = \pi(9)$ or 9π and $m\widehat{AD} = m\angle ABD$ or 40. Write a proportion to compare each part to its whole.



degree measure of arc → $\frac{\square}{\square}$ = $\frac{\square}{\square}$ ← arc length

degree measure of circumference → $\frac{\square}{\square}$ = $\frac{\square}{\square}$ ←

whole circle

Now solve the proportion for ℓ.

$\frac{\square}{\square} = \frac{\square}{\square}$

$\frac{40}{360}\square = \ell$ Multiply each side by 9π.

$\square = \ell$ Simplify.

The length of $\widehat{AD}$ is $\square$ units or about $\square$ units.

KEY CONCEPT

Arc Length Suppose a circle has radius r and circumference C. If an arc of the circle has degree measure A and length ℓ, then $\frac{A}{360} = \frac{\ell}{2\pi r}$ and $\frac{A}{360} \cdot C = \ell$.

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

Check Your Progress In $\odot A$, $AY = 21$ and $m\angle XAY = 45$. Find the length of $\widehat{WX}$.