Spring 2010 Practice Final Exam

Name_____

Convert the equation to the standard form for a hyperbola by completing the square on x and y.

1) $9x^2 - 4y^2 - 18x + 16y - 43 = 0$ 1) _____

A) $\frac{(x-1)^2}{4} - \frac{(y-2)^2}{9} = 1$

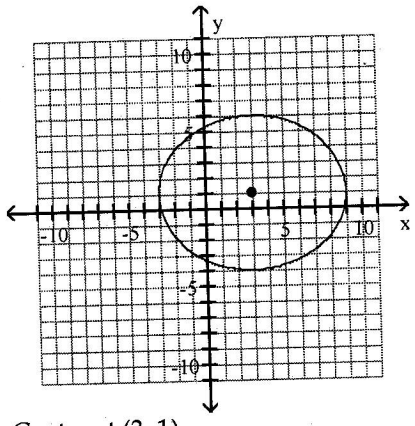
B) $\frac{(x+1)^2}{4} - \frac{(y-2)^2}{9} = 1$

C) $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{9} = 1$

D) $\frac{(x-1)^2}{9} - \frac{(y-2)^2}{4} = 1$

Find the standard form of the equation of the ellipse and give the location of its foci.

2) 2) _____



Center at (3, 1)

A) $\frac{(x-3)^2}{25} + \frac{(y-1)^2}{36} = 1$

foci at $(1 + \sqrt{11}, 3)$ and $(1 - \sqrt{11}, 3)$

B) $\frac{(x-1)^2}{36} + \frac{(y-3)^2}{25} = 1$

foci at $(3 + \sqrt{11}, 3)$ and $(3 - \sqrt{11}, 3)$

C) $\frac{(x-1)^2}{25} + \frac{(y-3)^2}{36} = 1$

foci at $(-\sqrt{11}, 1)$ and $(\sqrt{11}, 1)$

D) $\frac{(x-3)^2}{36} + \frac{(y-1)^2}{25} = 1$

foci at $(3 + \sqrt{11}, 1)$ and $(3 - \sqrt{11}, 1)$

Find the standard form of the equation of the ellipse satisfying the given conditions.

3) Major axis horizontal with length 8; length of minor axis = 4; center (0, 0) 3) _____

A) $\frac{x^2}{16} + \frac{y^2}{4} = 1$

B) $\frac{x^2}{64} + \frac{y^2}{16} = 1$

C) $\frac{x^2}{8} + \frac{y^2}{4} = 1$

D) $\frac{x^2}{4} + \frac{y^2}{16} = 1$

Find the standard form of the equation of the hyperbola satisfying the given conditions.

4) Endpoints of transverse axis: (0, –4), (0, 4); asymptote: $y = \frac{2}{3}x$ 4) _____

A) $\frac{y^2}{16} - \frac{x^2}{9} = 1$

B) $\frac{y^2}{36} - \frac{x^2}{16} = 1$

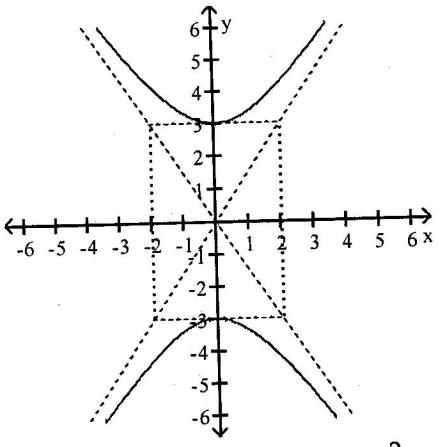
C) $\frac{y^2}{16} - \frac{x^2}{36} = 1$

D) $\frac{y^2}{9} - \frac{x^2}{4} = 1$

Find the standard form of the equation of the hyperbola.

5) _____

5)



A) $\frac{x^2}{9} - \frac{y^2}{4} = 1$ B) $\frac{x^2}{4} - \frac{y^2}{9} = 1$ C) $\frac{y^2}{4} - \frac{x^2}{9} = 1$ D) $\frac{y^2}{9} - \frac{x^2}{4} = 1$

Find the vertex, focus, and directrix of the parabola with the given equation.

6) _____

6) $(x - 4)^2 = 12(y + 2)$

A) vertex: (4, -2)
focus: (4, -5)
directrix: $x = 1$

B) vertex: (-4, 2)
focus: (-4, 5)
directrix: $y = -1$

C) vertex: (4, -2)
focus: (4, 1)
directrix: $y = -5$

D) vertex: (-2, 4)
focus: (-2, 7)
directrix: $y = 1$

Identify the equation without completing the square.

7) _____

7) $2x^2 - 4x + y - 2 = 0$

A) parabola B) hyperbola C) circle D) ellipse

Solve the problem.

8) _____

8) Find out how long it takes a $3500 investment to double if it is invested at 7% compounded quarterly. Round to the nearest tenth of a year. Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

A) 10.4 years B) 10.2 years C) 9.8 years D) 10 years

Evaluate or simplify the expression without using a calculator.

9) _____

9) $\ln \sqrt[3]{e}$

A) 3e B) $\frac{1}{3}$ C) 3 D) $\frac{e}{3}$

Solve the logarithmic equation. Be sure to reject any value that is not in the domain of the original logarithmic expressions. Give the exact answer.

10) _____

10) $\log_4 x + \log_4 (x - 15) = 2$

A) {4} B) {16} C) {-1, 16} D) {1, -16}

11) $\log_3 (x + 4) = 4 + \log_3 (x - 4)$ 11) _____

A) $\left\{\frac{41}{10}\right\}$ B) $\left\{-\frac{1}{10}\right\}$ C) $\left\{\frac{1}{10}\right\}$ D) $\left\{-\frac{41}{10}\right\}$

Evaluate the expression without using a calculator.

12) $\log_6 \frac{1}{\sqrt{6}}$ 12) _____

A) $-\frac{1}{2}$ B) $-\frac{1}{6}$ C) $\frac{1}{6}$ D) $\frac{1}{2}$

Solve.

13) The function $A = A_0 e^{-0.0077x}$ models the amount in pounds of a particular radioactive material stored in a concrete vault, where x is the number of years since the material was put into the vault. If 300 pounds of the material are initially put into the vault, how many pounds will be left after 130 years? 13) _____

A) 104 pounds B) 217 pounds C) 110 pounds D) 186 pounds

Solve the exponential equation. Use a calculator to obtain a decimal approximation, correct to two decimal places, for the solution.

14) $e^{5x - 9} - 4 = 1133$ 14) _____

A) 3.21 B) 0.61 C) 1.41 D) 2.41

Use properties of logarithms to expand the logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

15) $\log\left[\frac{7x^3\sqrt[4]{1 - x}}{2(x + 1)^2}\right]$ 15) _____

A) $\log 7 + \log x^3 + \log (1 - x)^{1/4} - \log 2 - \log (x + 1)^2$

B) $\log 7 + 3\log x + \frac{1}{4}\log (1 - x) - \log 2 - 2\log (x + 1)$

C) $\log (7x^3\sqrt[4]{1 - x}) - \log (2(x + 1)^2)$

D) $\log 7 + 3\log x + \frac{1}{4}\log (1 - x) - \log 2 + 2\log (x + 1)$

Solve the exponential equation. Express the solution set in terms of natural logarithms.

16) $4^{x + 4} = 5^{2x + 5}$ 16) _____

A) $\left\{\frac{5 \ln 5 - 4 \ln 4}{\ln 4 - 2 \ln 5}\right\}$ B) $\{\ln 5 - \ln 4\}$

C) $\{7 \ln 5 - 5 \ln 4\}$ D) $\left\{\ln\left[\frac{5^5}{4^4} - \frac{4}{5^2}\right]\right\}$

Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

17) $\frac{1}{4}[3\ln(x+2) - \ln x - \ln(x^2-2)]$ 17) _____

A) $\ln \sqrt[4]{\frac{(x+2)^3}{x(x^2-2)}}$

B) $\ln \sqrt[4]{\frac{x(x+2)^3}{(x^2-2)}}$

C) $\ln \sqrt[4]{\frac{3(x+2)}{x(x^2-2)}}$

D) $\ln \sqrt[4]{\frac{(x+2)^3(x^2-2)}{x}}$

Solve the equation by expressing each side as a power of the same base and then equating exponents.

18) $25^{x+3} = 125^{x-10}$ 18) _____

A) {33} B) {16} C) {36} D) {13}

Solve the system by the addition method.

19) $x^2 + y^2 = 9$
$9x^2 + 25y^2 = 225$ 19) _____

A) {(5, 0), (−5, 0)} B) {(3, 0), (−3, 0)} C) {(0, 5), (0, −5)} D) {(0, 3), (0, −3)}

Write the partial fraction decomposition of the rational expression.

20) $\frac{x+2}{x^3-2x^2+x}$ 20) _____

A) $\frac{2}{x} + \frac{-2}{x-1} + \frac{3}{(x-1)^2}$

B) $\frac{2}{x} + \frac{3}{x-1} + \frac{-2}{(x-1)^2}$

C) $\frac{2}{x} + \frac{-2}{x-1} + \frac{5}{(x-1)^2}$

D) $\frac{-2}{x} + \frac{2}{x-1} + \frac{3}{(x-1)^2}$

Solve the system by the substitution method.

21) $xy = 1$
$-28x - y = 11$ 21) _____

A) $\left\{\left(-\frac{1}{4}, -4\right)\right\}$

B) $\left\{\left(-\frac{1}{7}, -7\right), \left(-\frac{1}{4}, -4\right)\right\}$

C) {(7, −7), (4, −4)}

D) $\left\{\left(-7, -\frac{1}{7}\right), \left(-4, -\frac{1}{4}\right)\right\}$

Solve by the method of your choice.

22) $x^3 + y = 0$
$11x^2 - y = 0$ 22) _____

A) {(−1, 1), (−11, 1331)}

B) {(0, 0), (−11, 1331)}

C) {(0, 0), (−11, 121)}

D) {(0, 0), (11, −1331)}

Determine whether the equation defines y as a function of x.

23) $2x + 2y = 6$ 23) _____

A) y is a function of x

B) y is not a function of x

Evaluate the function at the given value of the independent variable and simplify.

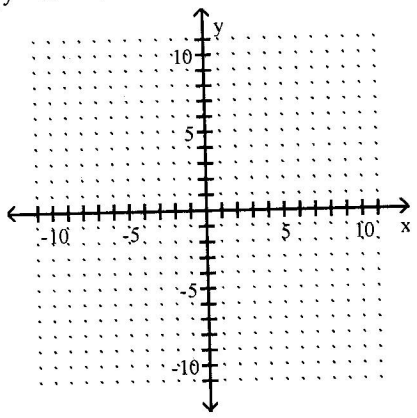
24) $f(x) = 4x^2 + 2x + 6$; $f(x - 1)$ 24) _____

A) $4x^2 - 6x + 12$ B) $-6x^2 + 4x + 8$ C) $4x^2 + 26x + 12$ D) $4x^2 - 6x + 8$

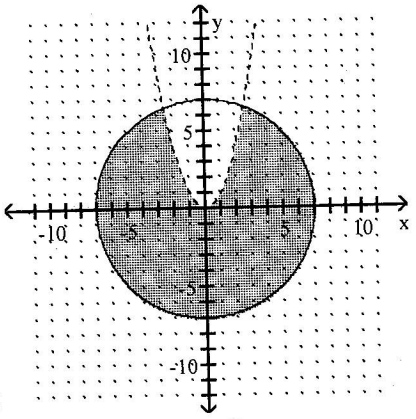
Graph the solution set of the system of inequalities or indicate that the system has no solution.

25) $x^2 + y^2 \leq 49$ 25) _____

$y - x^2 > 0$



A)



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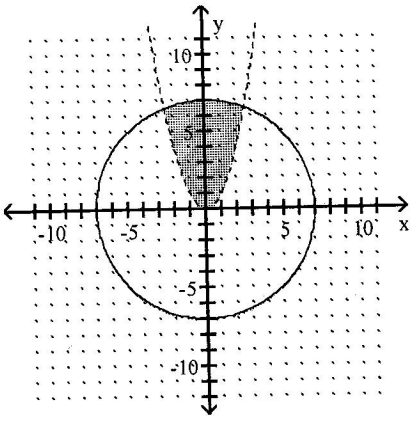

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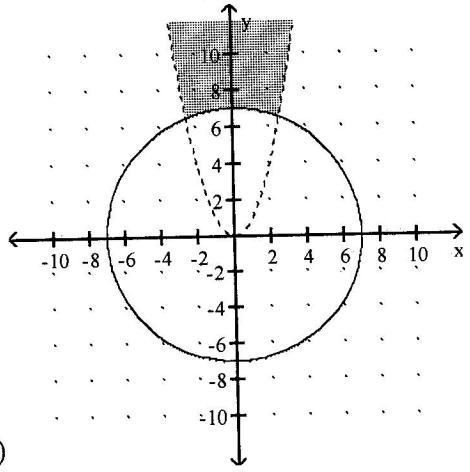
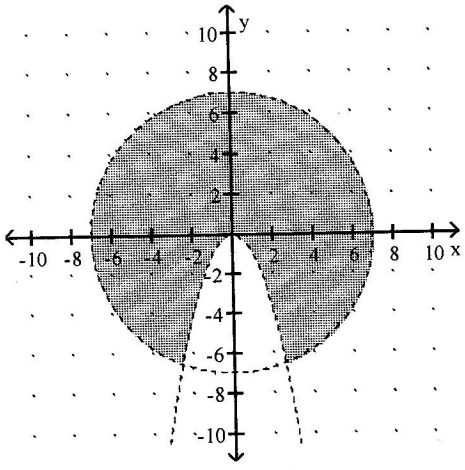


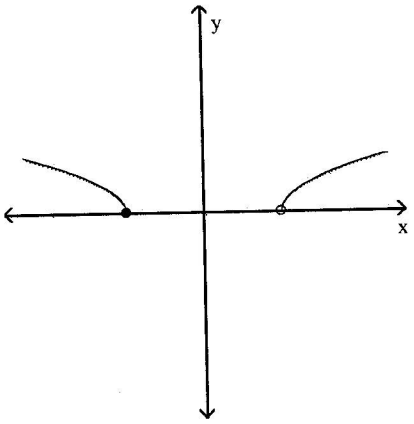
D)



Use the vertical line test to determine whether or not the graph is a graph in which y is a function of x.

26) _____

26)



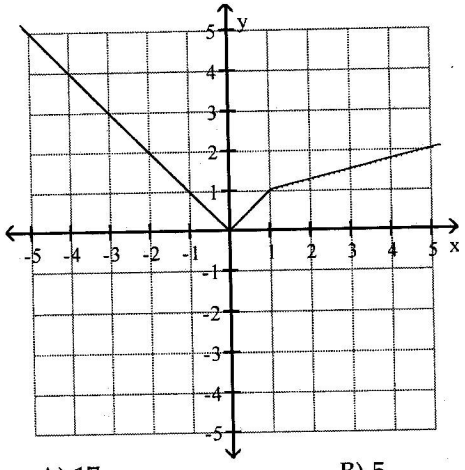
A) function

B) not a function

Use the graph to find the indicated function value.

27) _____

27) $y = f(x)$. Find $f(-5)$



A) 17

B) 5

C) 2

D) -5

Determine whether the given function is even, odd, or neither.

28) _____

28) $f(x) = x^3 + x^2 - 4$

A) Neither

B) Odd

C) Even

Evaluate the piecewise function at the given value of the independent variable.

29) _____

29) $f(x) = \begin{cases} x + 3 & \text{if } x > -2 \\ -(x + 3) & \text{if } x \le -2 \end{cases}$; $f(-6)$

A) 18

B) 3

C) -6

D) -3

Find and simplify the difference quotient $\frac{f(x + h) - f(x)}{h}$, $h \neq 0$ for the given function.

30) _____

30) $f(x) = 6x + 7$

A) 0

B) $6 + \frac{12(x + 7)}{h}$

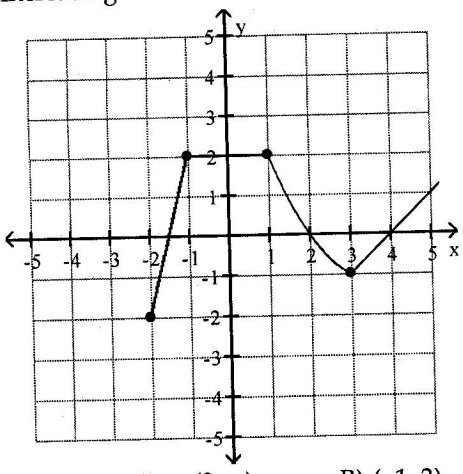
C) $6 + \frac{14}{h}$

D) 6

Identify the intervals where the function is changing as requested.

31) _____

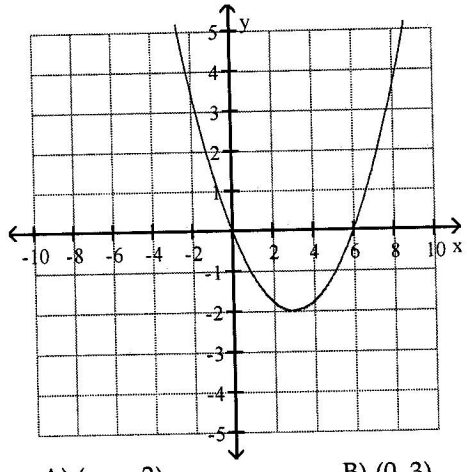
31) Increasing



A) (-2, -1) or (3, ∞) B) (-1, 3) C) (-2, 1) D) (-1, ∞)

32) _____

32) Decreasing



A) (-∞, -2) B) (0, 3) C) (0, -2) D) (-∞, 3)

Use the given conditions to write an equation for the line in point-slope form.

33) _____

33) Passing through (4, 2) and (3, 7)

A) $y - 2 = -5(x - 4)$ or $y - 7 = -5(x - 3)$
B) $y - 2 = 4(x + 4)$ or $y - 7 = 3(x - 2)$
C) $y - 2 = -5(x - 3)$ or $y - 7 = -5(x - 4)$
D) $y + 2 = -5(x + 4)$ or $y + 7 = -5(x + 3)$

Given functions f and g, perform the indicated operations.

34) _____

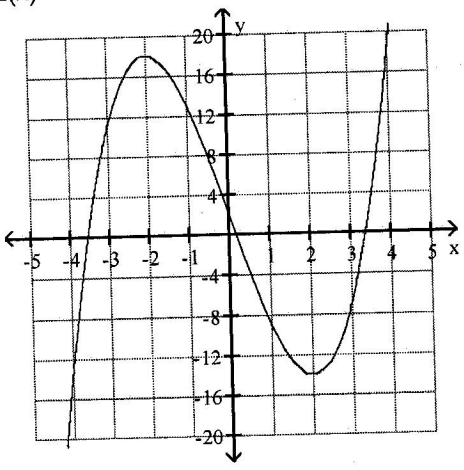
34) $f(x) = 7x - 9$, $g(x) = 2x - 4$

Find $f - g$.

A) $5x - 5$ B) $9x - 13$ C) $5x - 13$ D) $-5x + 5$

Use the graph of the given function to find any relative maxima and relative minima.

35) $f(x) = x^3 - 12x + 2$ 35) _____



A) maximum: (2, –14); minimum: (–2, 18)
B) no maximum or minimum
C) maximum: (–2, 18) and (0, 0); minimum: (2, –14)
D) minimum: (2, –14); maximum: (–2, 18)

Find the domain of the function.

36) $g(x) = \dfrac{3x}{x^2 - 81}$ 36) _____

A) $(-\infty, 0) \cup (0, \infty)$
B) $(81, \infty)$
C) $(-\infty, -9) \cup (-9, 9) \cup (9, \infty)$
D) $(-\infty, \infty)$

37) $f(x) = \sqrt{6 - x}$ 37) _____

A) $(-\infty, \sqrt{6}]$
B) $(-\infty, \sqrt{6}) \cup (\sqrt{6}, \infty)$
C) $(-\infty, 6) \cup (6, \infty)$
D) $(-\infty, 6]$

For the given functions f and g , find the indicated composition.

38) $f(x) = \dfrac{4}{x + 6}$, $g(x) = \dfrac{7}{8x}$ 38) _____

$(f \circ g)(x)$

A) $\dfrac{32x}{7 - 48x}$ B) $\dfrac{4x}{7 + 48x}$ C) $\dfrac{7x + 42}{32x}$ D) $\dfrac{32x}{7 + 48x}$

Determine which two functions are inverses of each other.

39) $f(x) = \dfrac{x - 6}{4}$ $g(x) = 4x - 6$ $h(x) = \dfrac{x + 6}{4}$ 39) _____

A) f(x) and h(x) B) f(x) and g(x) C) g(x) and h(x) D) None

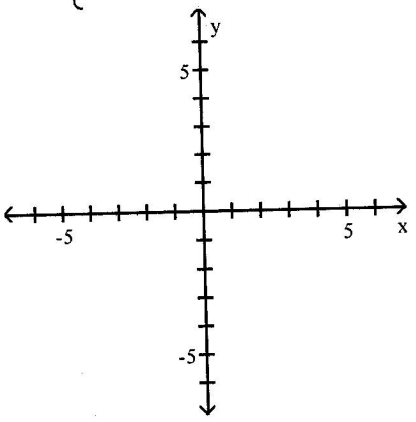
Find the inverse of the one-to-one function.

40) $f(x) = \sqrt[3]{x - 6}$ 40) _____

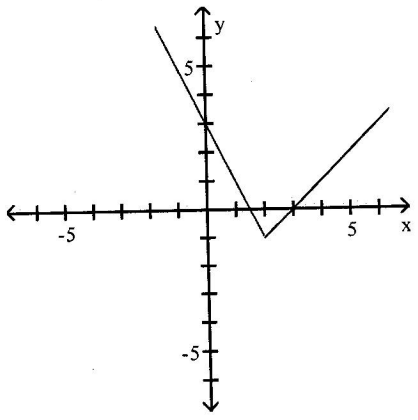
A) $f^{-1}(x) = x^3 + 6$ B) $f^{-1}(x) = x + 6$ C) $f^{-1}(x) = x^3 + 36$ D) $f^{-1}(x) = \dfrac{1}{x^3 + 6}$

Graph the function.

41) $f(x) = \begin{cases} -x + 3 & \text{if } x < 2 \\ 2x - 3 & \text{if } x \geq 2 \end{cases}$ 41) _____



A)



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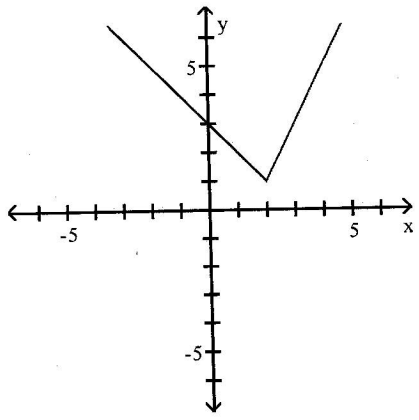
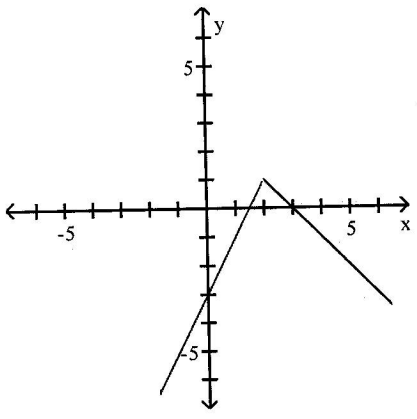
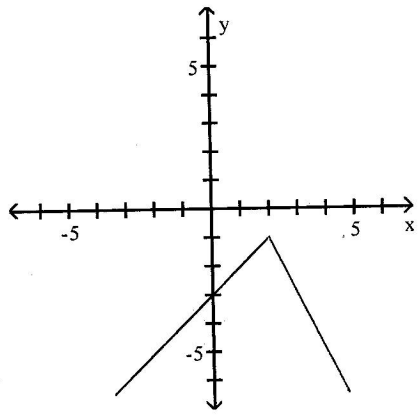


C)



D)



Determine whether the given quadratic function has a minimum value or maximum value. Then find the coordinates of the minimum or maximum point.

42) $f(x) = 5x^2 - 10x$ 42) _____

A) minimum; $(-1, -5)$
B) maximum; $(-1, -5)$
C) minimum; $(1, -5)$
D) maximum; $(1, -5)$

Find the coordinates of the vertex for the parabola defined by the given quadratic function.

43) $f(x) = -x^2 + 10x - 2$ 43) _____

A) (5, 23)
B) (10, -2)
C) (-5, -77)
D) (-5, -27)

Find the x-intercepts (if any) for the graph of the quadratic function.

44) $f(x) = x^2 + 14x + 35$ Give your answers in exact form. 44) _____

A) $(-14 \pm \sqrt{35}, 0)$ B) $(7 + \sqrt{14}, 0)$ C) $(-7 \pm \sqrt{14}, 0)$ D) $(7 \pm \sqrt{35}, 0)$

Find the y-intercept for the graph of the quadratic function.

45) $f(x) = -x^2 + 2x + 3$ 45) _____

A) $(0, -3)$ B) $(0, 3)$ C) $(0, -1)$ D) $(3, 0)$

Find the domain and range of the quadratic function whose graph is described.

46) The vertex is $(1, 0)$ and the graph opens down. 46) _____

A) Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

B) Domain: $(-\infty, 1]$
Range: $(-\infty, 0]$

C) Domain: $(-\infty, \infty)$
Range: $(-\infty, 0]$

D) Domain: $(-\infty, \infty)$
Range: $(-\infty, 1]$

Use the Leading Coefficient Test to determine the end behavior of the polynomial function.

47) $f(x) = 2x^4 - 3x^3 + 3x^2 + 5x - 2$ 47) _____

A) rises to the left and falls to the right
B) falls to the left and falls to the right
C) rises to the left and rises to the right
D) falls to the left and rises to the right

Find the zeros of the polynomial function.

48) $f(x) = x^3 - 5x^2 - 4x + 20$ 48) _____

A) $x = 5, x = -2, x = 2$
B) $x = -5, x = -2, x = 2$
C) $x = -2, x = 2$
D) $x = 5, x = 4$

Find the zeros for the polynomial function and give the multiplicity for each zero. State whether the graph crosses the x-axis or touches the x-axis and turns around, at each zero.

49) $f(x) = 3(x^2 + 1)(x - 2)^2$ 49) _____

A) -1, multiplicity 1, crosses the x-axis; 2, multiplicity 2, crosses the x-axis
B) 2, multiplicity 2, touches the x-axis and turns around
C) 2, multiplicity 2, crosses the x-axis
D) -1, multiplicity 1, crosses the x-axis; 2, multiplicity 2, touches the x-axis and turns around.

Solve the problem.

50) Solve the equation $3x^3 - 32x^2 + 73x + 28 = 0$ given that 4 is a zero of $f(x) = 3x^3 - 32x^2 + 73x + 28$. 50) _____

A) $\left\{4, -1, \frac{7}{3}\right\}$ B) $\left\{4, 1, -\frac{7}{3}\right\}$ C) $\left\{4, -7, \frac{1}{3}\right\}$ D) $\left\{4, 7, -\frac{1}{3}\right\}$

Divide using long division.

51) $\dfrac{x^4 + 3x^3 + 6x^2 + 5x + 9}{x^2 + 2x + 3}$ 51) _____

A) $x^2 + x + 1 + \dfrac{6}{x^2 + 2x + 3}$

B) $x^2 + 4x + 17$

C) $x^2 + 4x + 17 + \dfrac{49x + 48}{x^2 + 2x + 3}$

D) $x^2 + x + 1$

Divide using synthetic division.

52) $\dfrac{-4x^3 - 16x^2 + 18x - 10}{x + 5}$ 52) _____

A) $-4x^2\,x - \dfrac{16}{5} - 2$

B) $-4x^2 + 4x - 2$

C) $-\dfrac{4}{5}x^2 - \dfrac{16}{5}x + \dfrac{18}{5}$

D) $4x^2 - 5x - 2$

Find an nth degree polynomial function with real coefficients satisfying the given conditions.

53) n = 3; -2 and -3 + 2i are zeros; leading coefficient is 1 53) _____

A) $f(x) = x^3 + 8x^2 + 25x + 26$

B) $f(x) = x^3 + 8x^2 + 15x + 26$

C) $f(x) = x^3 + 5x^2 + 25x - 14$

D) $f(x) = x^3 - 4x^2 + 25x + 26$

Use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for the given function.

54) $f(x) = x^7 + x^6 + x^2 + x + 2$ 54) _____

A) 0 positive zeros, 2 or 0 negative zeros

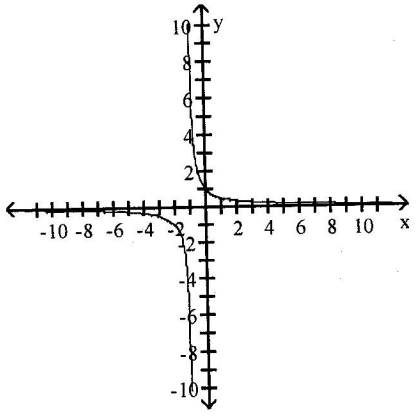
B) 0 positive zeros, 1 negative zero

C) 0 positive zeros, 3 or 1 negative zeros

D) 0 positive zeros, 0 negative zeros

Use the graph of the rational function shown to complete the statement.

55) 55) _____



As $x \to -1^-$, $f(x) \to$?

A) $+\infty$

B) 0

C) $-\infty$

D) 1

Find the vertical asymptotes, if any, of the graph of the rational function.

56) $g(x) = \dfrac{x}{x^2 - 25}$ 56) _____

A) x = 5, x = -5, x = 0

B) x = 5

C) x = 5, x = -5

D) no vertical asymptote

Find the horizontal asymptote, if any, of the graph of the rational function.

57) $g(x) = \dfrac{4x^2 - 2x - 8}{6x^2 - 7x + 5}$ 57) _____

A) y = 0

B) $y = \dfrac{2}{7}$

C) $y = \dfrac{2}{3}$

D) no horizontal asymptote

Find the slant asymptote, if any, of the graph of the rational function.

58) $f(x) = \frac{x^2 + 9x - 3}{x - 5}$ 58) _____

A) $y = x + 9$
B) $y = x$
C) $y = x + 14$
D) no slant asymptote

Evaluate the factorial expression.

59) $\frac{n(n+2)!}{(n+3)!}$ 59) _____

A) $\frac{n}{(n+3)!}$ B) $\frac{n}{n+3}$ C) $\frac{n}{3}$ D) $\frac{1}{n+3}$

Use the formula for the sum of the first n terms of an arithmetic sequence to find the indicated sum.

60) $\sum_{i=1}^{45} (-2i - 6)$ 60) _____

A) -2340 B) -2182.5 C) -2295 D) -2025

Express the sum using summation notation. Use 1 as the lower limit of summation and i for the index of summation.

61) $a + 1 + \frac{a+2}{2} + \ldots + \frac{a+5}{5}$ 61) _____

A) $\sum_{i=1}^{5} \frac{a+i}{i}$ B) $\sum_{i=1}^{n} \frac{a+i}{i}$ C) $\sum_{i=0}^{5} \frac{a+i}{i}$ D) $\sum_{i=0}^{n} \frac{a+i}{i}$

Write the first five terms of the arithmetic sequence.

62) $a_n = a_{n-1} + 4;\ a_1 = -5$ 62) _____

A) 4, -1, -6, -11, -16
B) -5, 4, -1, 3, 7
C) -6, -2, 2, 6, 10
D) -5, -1, 3, 7, 11

Use the formula for the general term (the nth term) of an arithmetic sequence to find the indicated term of the sequence with the given first term, a_1, and common difference, d.

63) Find a_{80} when $a_1 = -7$, $d = 5$. 63) _____

A) -407 B) 388 C) -402 D) 393

Use the formula for the general term (the nth term) of a geometric sequence to find the indicated term of the sequence with the given first term, a_1, and common ratio, r.

64) Find a_{11} when $a_1 = 4$, $r = -2$. 64) _____

A) 4100 B) -16 C) 4096 D) -8192

Solve the problem. Round to the nearest dollar if needed.

65) Kurt deposits $150 each month into an account paying annual interest of 6.5% compounded monthly. How much will his account have in it at the end of 5 years? 65) _____

A) $854 B) $10,730 C) $10,447 D) $10,601