

Trigonometry Final Exam Review

Name _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The given angle is in standard position. Determine the quadrant in which the angle lies.

1) 26°

1) _____

A) Quadrant II

B) Quadrant III

C) Quadrant I

D) Quadrant IV

Convert the angle in degrees to radians. Express answer as a multiple of π .

2) 75°

2) _____

A) $\frac{5\pi}{12}$ radians

B) $\frac{6\pi}{13}$ radians

C) $\frac{1}{3}\pi$ radians

D) $\frac{4\pi}{11}$ radians

Convert the angle in radians to degrees. Round to two decimal places.

3) $\frac{4}{3}\pi$ radians

3) _____

A) 242°

B) 239°

C) 240°

D) 241°

Find a positive angle less than 360° or 2π that is coterminal with the given angle.

4) 421°

4) _____

A) 210.5°

B) 61°

C) 51°

D) 241°

The point $P(x, y)$ on the unit circle that corresponds to a real number t is given. Find the value of the indicated trigonometric function at t .

5) $\left(\frac{2}{5}, \frac{\sqrt{21}}{5}\right)$ Find $\tan t$.

5) _____

A) $\frac{5}{2}$

B) $\frac{\sqrt{21}}{5}$

C) $\frac{\sqrt{21}}{2}$

D) $\frac{2\sqrt{21}}{21}$

Find the exact value of the trigonometric function. Do not use a calculator.

6) $\csc \frac{\pi}{4}$

6) _____

A) $\frac{\sqrt{2}}{2}$

B) $\sqrt{3}$

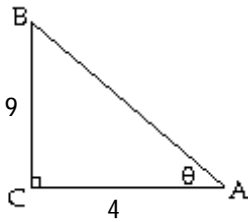
C) $\frac{\sqrt{3}}{2}$

D) $\sqrt{2}$

Use the Pythagorean Theorem to find the length of the missing side. Then find the indicated trigonometric function of the given angle. Give an exact answer with a rational denominator.

7) Find $\sin \theta$.

7) _____



A) $\frac{4\sqrt{97}}{97}$

B) $\frac{9\sqrt{97}}{97}$

C) $\frac{\sqrt{97}}{9}$

D) $\frac{\sqrt{97}}{4}$

Solve the problem.

8) A surveyor is measuring the distance across a small lake. He has set up his transit on one side of the lake 110 feet from a piling that is directly across from a pier on the other side of the lake. From his transit, the angle between the piling and the pier is 65° . What is the distance between the piling and the pier to the nearest foot?

8) _____

A) 100 feet

B) 236 feet

C) 46 feet

D) 51 feet

9) A building 170 feet tall casts a 60 foot long shadow. If a person looks down from the top of the building, what is the measure of the angle between the end of the shadow and the vertical side of the building (to the nearest degree)? (Assume the person's eyes are level with the top of the building.)

9) _____

A) 21°

B) 19°

C) 71°

D) 69°

A point on the terminal side of angle θ is given. Find the exact value of the indicated trigonometric function of θ .

10) $(4, -3)$ Find $\sin \theta$.

10) _____

A) $\frac{5}{4}$

B) $\frac{4}{5}$

C) $-\frac{3}{5}$

D) -3

11) $(-\frac{1}{2}, \frac{1}{3})$ Find $\cos \theta$.

11) _____

A) $-\frac{13}{2}$

B) $\frac{2\sqrt{13}}{13}$

C) $-\frac{3\sqrt{13}}{13}$

D) $\frac{13}{3}$

Find the exact value of the indicated trigonometric function of θ .

12) $\sin \theta = -\frac{8}{9}$, $\tan \theta > 0$ Find $\sec \theta$.

12) _____

A) $-\frac{8\sqrt{17}}{17}$

B) $\frac{\sqrt{9}}{8}$

C) $-\frac{\sqrt{17}}{9}$

D) $-\frac{9\sqrt{17}}{17}$

Find the reference angle for the given angle.

13) -245°

13) _____

A) 155°

B) 65°

C) 25°

D) 115°

Use reference angles to find the exact value of the expression. Do not use a calculator.

14) $\sin \frac{5\pi}{3}$

14) _____

A) -1

B) $-\frac{1}{2}$

C) $-\frac{\sqrt{3}}{2}$

D) $\frac{\sqrt{3}}{2}$

Determine the amplitude or period as requested.

15) Period of $y = \frac{7}{4} \sin \frac{4\pi}{3}x$

15) _____

A) $\frac{3}{2}$

B) $\frac{2}{7}$

C) $\frac{8\pi}{3}$

D) $\frac{7\pi}{2}$

Determine the phase shift of the function.

16) $y = 4 \cos (4\pi x - 5)$

16) _____

A) 5 units to the left

B) $\frac{5}{4\pi}$ units to the right

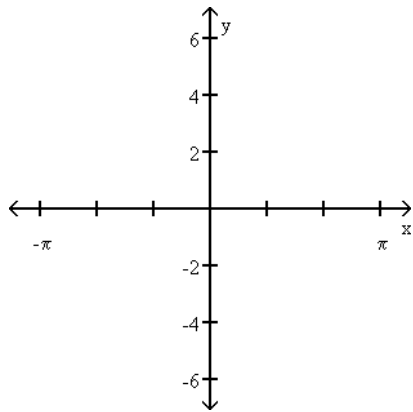
C) 5 units to the right

D) $\frac{5}{4}$ units to the left

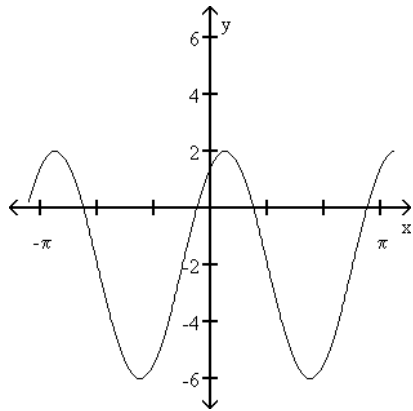
Use a vertical shift to graph the function.

$$17) y = 4 \sin\left(2x + \frac{\pi}{3}\right) - 2$$

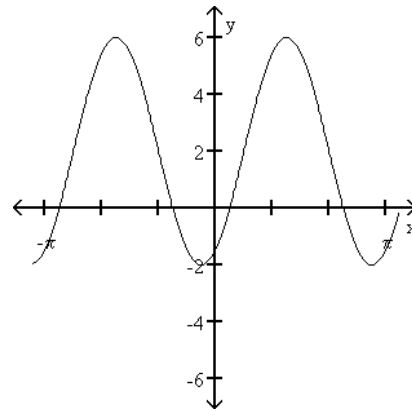
17) _____



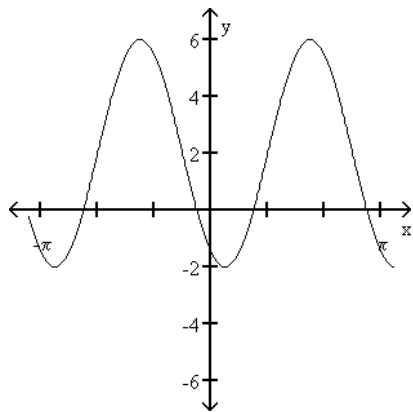
A)



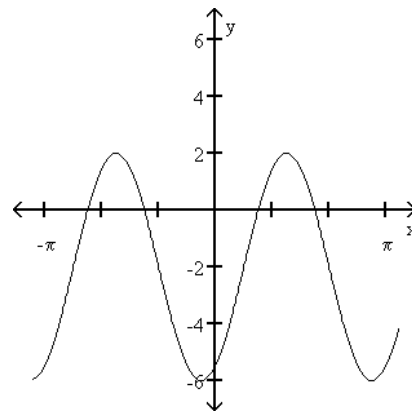
B)



C)

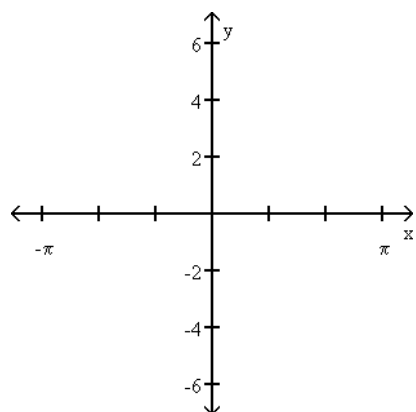


D)

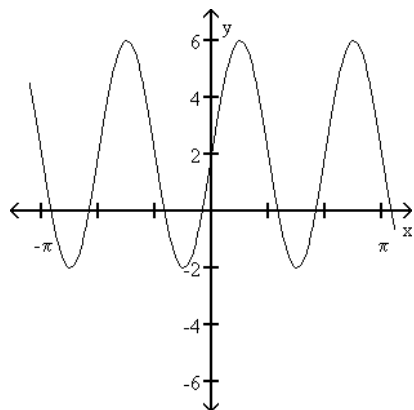


18) $y = -4 \cos\left(3x - \frac{\pi}{2}\right) + 2$

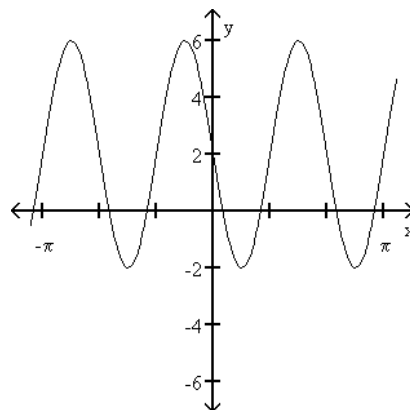
18) _____



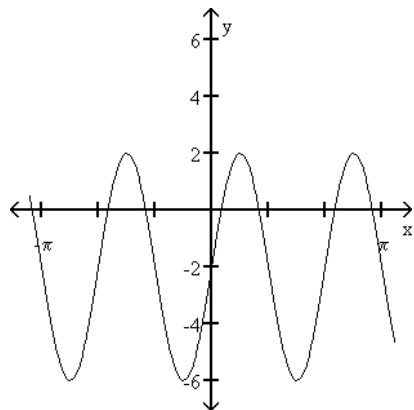
A)



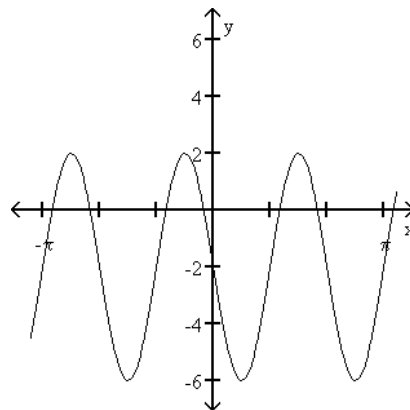
B)



C)



D)



Find the exact value of the expression.

19) $\sin^{-1} \frac{\sqrt{2}}{2}$

19) _____

A) $\frac{3\pi}{4}$

B) $\frac{\pi}{3}$

C) $\frac{\pi}{4}$

D) $\frac{2\pi}{3}$

20) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ 20) _____

- A) $\frac{2\pi}{3}$ B) $\frac{\pi}{3}$ C) $\frac{5\pi}{6}$ D) $\frac{\pi}{6}$

21) $\tan^{-1} 1$ 21) _____

- A) $\frac{5\pi}{4}$ B) $\frac{\pi}{4}$ C) $\frac{2\pi}{3}$ D) $\frac{\pi}{3}$

Use a calculator to find the value of the expression rounded to two decimal places.

22) $\sin^{-1}\left(\frac{\sqrt{2}}{5}\right)$ 22) _____

- A) 1.28 B) 16.43 C) 0.29 D) 73.57

Solve the problem.

23) A radio transmission tower is 240 feet tall. How long should a guy wire be if it is to be attached 11 feet from the top and is to make an angle of 22° with the ground? Give your answer to the nearest tenth of a foot. 23) _____

- A) 611.3 feet B) 258.8 feet C) 640.7 feet D) 247.0 feet

24) A straight trail with a uniform inclination of 17° leads from a lodge at an elevation of 700 feet to a mountain lake at an elevation of 9100 feet. What is the length of the trail (to the nearest foot)? 24) _____

- A) 28,731 feet B) 8784 feet C) 31,125 feet D) 9516 feet

Find all solutions of the equation.

25) $10 \cos x - 4\sqrt{2} = 8 \cos x - 3\sqrt{2}$ 25) _____

- A) $x = \frac{3\pi}{4} + n\pi$ or $x = \frac{5\pi}{4} + n\pi$ B) $x = \frac{\pi}{4} + 2n\pi$ or $x = \frac{7\pi}{4} + 2n\pi$
 C) $x = \frac{\pi}{4} + n\pi$ or $x = \frac{7\pi}{4} + n\pi$ D) $x = \frac{3\pi}{4} + 2n\pi$ or $x = \frac{5\pi}{4} + 2n\pi$

Solve the equation on the interval $[0, 2\pi)$.

26) $\cos 2x = \frac{\sqrt{3}}{2}$

26) _____

A) $\frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

B) $\frac{\pi}{2}$

C) $\frac{\pi}{6}, \frac{11\pi}{6}$

D) $\frac{3\pi}{2}$

27) $2 \sin^2 x = \sin x$

27) _____

A) $\frac{\pi}{3}, \frac{2\pi}{3}$

B) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$

C) $\frac{\pi}{6}, \frac{5\pi}{6}$

D) $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

Solve the equation on the interval $[0, 2\pi)$.

28) $\sin x - 2 \sin x \cos x = 0$

28) _____

A) $\frac{\pi}{3}, \frac{5\pi}{3}$

B) $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

C) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

D) $\frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$

Solve the triangle. Round lengths to the nearest tenth and angle measures to the nearest degree.

29) $B = 14^\circ$
 $C = 100^\circ$
 $b = 29$

29) _____

A) $A = 66^\circ, a = 111.5, c = 120.1$

B) $A = 66^\circ, a = 109.5, c = 118.1$

C) $A = 64^\circ, a = 120.1, c = 111.5$

D) $A = 64^\circ, a = 118.1, c = 109.5$

Two sides and an angle (SSA) of a triangle are given. Determine whether the given measurements produce one triangle, two triangles, or no triangle at all. Solve each triangle that results. Round lengths to the nearest tenth and angle measures to the nearest degree.

30) $B = 31^\circ, b = 8, a = 32$

30) _____

A) $A = 29^\circ, C = 119^\circ, c = 40$

B) $A = 30^\circ, C = 120^\circ, c = 41.5$

C) no triangle

D) $A = 27^\circ, C = 121^\circ, c = 37$

31) $B = 13^\circ, b = 10.4, a = 23.12$

31) _____

A) $A = 30^\circ, C = 137^\circ, c = 31.5$

B) $A = 150^\circ, C = 17^\circ, c = 13.5$

C) $A_1 = 30^\circ, C_1 = 137^\circ, c_1 = 31.5;$

D) no triangle

$A_2 = 150^\circ, C_2 = 17^\circ, c_2 = 13.5$

Find the area of the triangle having the given measurements. Round to the nearest square unit.

32) $A = 24^\circ$, $b = 20$ meters, $c = 9$ meters

32) _____

- A) 76 square meters
C) 37 square meters

- B) 74 square meters
D) 19 square meters

Solve the triangle. Round lengths to the nearest tenth and angle measures to the nearest degree.

33) $a = 6$, $c = 12$, $B = 123^\circ$

33) _____

- A) $b = 21.9$, $A = 16^\circ$, $C = 41^\circ$
C) $b = 16.1$, $A = 18^\circ$, $C = 39^\circ$

- B) $b = 19$, $A = 20^\circ$, $C = 37^\circ$
D) no triangle

Use Heron's formula to find the area of the triangle. Round to the nearest square unit.

34) $a = 9$ inches, $b = 12$ inches, $c = 5$ inches

34) _____

- A) 24 square inches
C) 18 square inches

- B) 16 square inches
D) 26 square inches

Polar coordinates of a point are given. Find the rectangular coordinates of the point.

35) $(2, 270^\circ)$

35) _____

A) $(0, -2)$

B) $(0, 2)$

C) $(2, 0)$

D) $(-2, 0)$

36) $\left(-9, \frac{2\pi}{3}\right)$

36) _____

A) $\left(-\frac{9}{2}, \frac{-9\sqrt{3}}{2}\right)$

B) $\left(\frac{9}{2}, \frac{9\sqrt{3}}{2}\right)$

C) $\left(-\frac{9}{2}, \frac{9\sqrt{3}}{2}\right)$

D) $\left(\frac{9}{2}, \frac{-9\sqrt{3}}{2}\right)$

The rectangular coordinates of a point are given. Find polar coordinates of the point. Express θ in radians.

37) $(9, -9)$

37) _____

A) $\left(9, -\frac{7\pi}{4}\right)$

B) $\left(9, \frac{\pi}{4}\right)$

C) $\left(9\sqrt{2}, -\frac{7\pi}{4}\right)$

D) $\left(9\sqrt{2}, \frac{7\pi}{4}\right)$

38) $(4\sqrt{3}, 4)$

38) _____

A) $\left(8, \frac{\pi}{6}\right)$

B) $\left(4, \frac{\pi}{3}\right)$

C) $\left(4, \frac{\pi}{6}\right)$

D) $\left(8, \frac{\pi}{3}\right)$

Convert the rectangular equation to a polar equation that expresses r in terms of θ .

39) $y = 5$

39) _____

A) $r = \frac{5}{\cos \theta}$

B) $r = \frac{5}{\sin \theta}$

C) $r = 5$

D) $\sin \theta = 5$

40) $y^2 = 3x$

40) _____

A) $r = 3 \cot^2 x$

B) $r = 9 \cot x \csc x$

C) $r = 3 \cot x \csc x$

D) $r^2(\cos \theta + \sin \theta) = 3$

Convert the polar equation to a rectangular equation.

41) $r = 6$

41) _____

A) $x = 6$

B) $y = 6$

C) $y^2 = 36$

D) $x^2 + y^2 = 36$

42) $r \cos \theta = 6$

42) _____

A) $y^2 = 6$

B) $x^2 + y^2 = 6$

C) $x = 6$

D) $y = 6$

Write the complex number in polar form. Express the argument in degrees.

43) $-15 - 20i$

43) _____

A) $25(\cos 233.1^\circ + i \sin 233.1^\circ)$

B) $25(\cos 53.1^\circ + i \sin 53.1^\circ)$

C) $25(\cos 306.9^\circ + i \sin 306.9^\circ)$

D) $25(\cos 126.9^\circ + i \sin 126.9^\circ)$

Write the complex number in polar form. Express the argument in radians.

44) $-2 + 2\sqrt{3}i$

44) _____

A) $4 \left\{ \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right\}$

B) $2\sqrt{3} \left\{ \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right\}$

C) $4 \left\{ \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right\}$

D) $2\sqrt{3} \left\{ \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right\}$

Write the complex number in rectangular form.

45) $9(\cos 120^\circ + i \sin 120^\circ)$

45) _____

A) $-\frac{9}{2} + \frac{9\sqrt{3}}{2}i$

B) $\frac{9}{2} + \frac{9\sqrt{3}}{2}i$

C) $\frac{9}{2} + \frac{-9\sqrt{3}}{2}i$

D) $-\frac{9}{2} + \frac{-9\sqrt{3}}{2}i$

46) $-3\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

46) _____

A) $\frac{3}{2} + \frac{-3\sqrt{3}}{2}i$

B) $-\frac{3}{2} + \frac{3\sqrt{3}}{2}i$

C) $\frac{3}{2} + \frac{3\sqrt{3}}{2}i$

D) $-\frac{3}{2} + \frac{-3\sqrt{3}}{2}i$

Use DeMoivre's Theorem to find the indicated power of the complex number. Write the answer in rectangular form.

47) $[4(\cos 15^\circ + i \sin 15^\circ)]^4$

47) _____

A) $128\sqrt{3} + 128i$

B) $256i$

C) $128 + 128\sqrt{3}i$

D) $128 + 128i$

48) $(1 - i)^{10}$

48) _____

A) $32 - 32i$

B) $-32i$

C) 32

D) $-32 + 32i$

Find all the complex roots. Write the answer in the indicated form.

49) The complex cube roots of $27(\cos 102^\circ + i \sin 102^\circ)$ (polar form)

49) _____

A) $3(\cos 34^\circ + i \sin 34^\circ)$, $3(\cos 74^\circ + i \sin 74^\circ)$, $3(\cos 114^\circ + i \sin 114^\circ)$

B) $-3(\cos 34^\circ + i \sin 34^\circ)$, $3(\cos 154^\circ + i \sin 154^\circ)$, $-3(\cos 274^\circ + i \sin 274^\circ)$

C) $3(\cos 34^\circ + i \sin 34^\circ)$, $3(\cos 154^\circ + i \sin 154^\circ)$, $3(\cos 274^\circ + i \sin 274^\circ)$

D) $-3(\cos 34^\circ + i \sin 34^\circ)$, $3(\cos 74^\circ + i \sin 74^\circ)$, $-3(\cos 114^\circ + i \sin 114^\circ)$

50) The complex square roots of $4 + 4\sqrt{3}i$ (rectangular form)

50) _____

A) $-\sqrt{6} - \sqrt{2}i$, $\sqrt{6} - \sqrt{2}i$

B) $\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$, $-\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$

C) $\sqrt{6} + \sqrt{2}i$, $-\sqrt{6} - \sqrt{2}i$

D) $\sqrt{6} - \sqrt{2}i$, $-\sqrt{6} + \sqrt{2}i$