

# L7] Combinations of Functions; Composite Functions

Domain - set of x-values

ex)  $\frac{(\text{blah})^3}{\text{stuff}} \neq 0$ ;  $\sqrt{\text{stuff}} \geq 0$

ex 1)  $f(x) = x^2 + 3x - 17$

Find the domain.  
 $\rightarrow (-\infty, \infty)$  all real #s

ex 2)  $g(x) = \frac{5x}{x^2 - 49}$ . Domain?

$x^2 - 49 \neq 0$  diff of squares  
 $\frac{5x}{(x-7)(x+7)} \neq 0$  zero product prop.  
 $x \neq 7 \mid x \neq -7$

ex 3)  $f(x) = \frac{5x}{x^2 + 49}$ . Domain?

$x^2 + 49 \neq 0$   
 $x^2 \neq -49$   $\Rightarrow$

$\rightarrow (-\infty, \infty)$

ex 4)  $h(x) = \sqrt{9x - 27}$ . Domain?

$9x - 27 \geq 0$   
 $9x \geq 27$

$x \geq 3 \rightarrow [3, \infty)$

ex 5) Let  $f(x) = x - 5$  &  $g(x) = x^2 - 1$ . Find the following & state the domain.

f+g a)  $(f+g)(x) = f(x) + g(x)$   
 $= x - 5 + x^2 - 1 = x^2 + x - 6$

domain:  $(-\infty, \infty)$

b)  $(f-g)(x) = f(x) - g(x)$   
 $= x - 5 - (x^2 - 1) = x - 5 - x^2 + 1 = -x^2 + x - 4$

domain:  $(-\infty, \infty)$

c)  $(f \cdot g)(x) = f(x) \cdot g(x)$   
 $= (x - 5)(x^2 - 1) = x^3 - x - 5x^2 + 5 = x^3 - 5x^2 - x + 5$

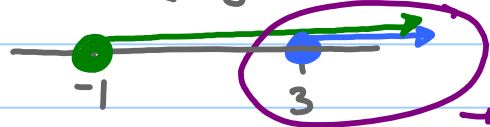
domain:  $(-\infty, \infty)$

d)  $(\frac{f}{g})(x) = \frac{f(x)}{g(x)} = \frac{x-5}{x^2-1} \neq 0 = \frac{x-5}{(x-1)(x+1)}$   
 $x \neq 1 \mid x \neq -1$  domain:  $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

ex 9) Let  $f(x) = \sqrt{x-3}$  and  $g(x) = \sqrt{x+1}$ .

Find  $(f+g)(x)$  & domain.

$\rightarrow (f+g)(x) = f(x) + g(x) = \sqrt{x-3} + \sqrt{x+1}$



domain must be true for both  
 $x-3 \geq 0 \rightarrow x \geq 3$   
 $x+1 \geq 0 \rightarrow x \geq -1$

and intersection domain:  $[3, \infty)$

Composite Functions - "stuff the turkey"

$(f \circ g)(x) = f(g(x))$        $(g \circ f)(x) = g(f(x))$

↑ turkey stuffing

ex 10) Let  $f(x) = 5x+6$  &  $g(x) = 2x^2-x-1$

a) Find  $(f \circ g)(x) = f(g(x))$   
 $= 5(2x^2-x-1) + 6 = 10x^2 - 5x - 5 + 6$   
 $= 10x^2 - 5x + 1$

b) Find  $(g \circ f)(x) = g(f(x))$   
 $= 2(5x+6)^2 - (5x+6) - 1$   
 $= 2(5x+6)(5x+6) - (5x+6) - 1$   
 $= 50x^2 + 120x + 72 - 5x - 6 - 1$   
 $= 50x^2 + 115x + 65$

c) Find  $f(g(-1)) = 10(-1)^2 - 5(-1) + 1 = 10 + 5 + 1 = 16$

ex 11) Let  $f(x) = \frac{4}{x+2}$  &  $g(x) = \frac{1}{x}$ . Find  $f(g(x))$  & the domain of  $f(g(x))$ .

$\rightarrow \frac{4}{(\frac{1}{x})+2} = \frac{4}{\frac{1}{x} + \frac{2 \cdot x}{1 \cdot x}} = \frac{4}{\frac{1+2x}{x}} \leftarrow \div \text{ mult by reciprocal}$   
 $= 4 \div \frac{1+2x}{x} = 4 \cdot \frac{x}{1+2x} = \frac{4x}{1+2x}$

$\therefore$  domain:  $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 0) \cup (0, \infty)$   
 $1+2x \neq 0$   
 $2x \neq -1$   
 $x \neq -\frac{1}{2}$

ex 12) Express  $h(x)$  as a composition of two functions.

$$h(x) = \sqrt{x^2 + 5}$$

→ Find  $f(g(x)) = h(x)$

$$f(x) = \sqrt{x}$$

$$g(x) = x^2 + 5$$

HW: p 229, # 2-78 EOE