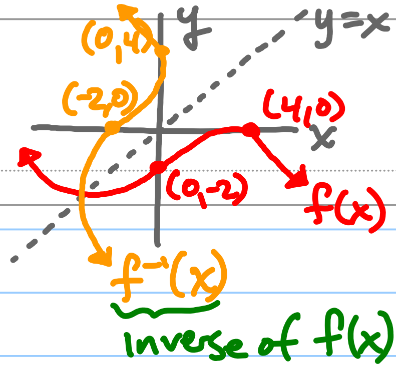
1/31 THU

1.8 Inverse Functions $f^{-1}(x)$



→ swap the domain & range
- switch x & y
- Isolate y

→ If $f(g(x)) = g(f(x)) = $ "x", then $f(x)$ & $g(x)$ are inverses

ex 1) Show that each function is the inverse of the other.

$f(x) = 4x-7$ & $g(x) = \frac{x+7}{4}$

→ $f(g(x)) = 4\left(\frac{x+7}{4}\right) - 7 = x+7-7 = x$

$g(f(x)) = \frac{(4x-7)+7}{4} = \frac{4x}{4} = x$

Since $f(g(x)) = g(f(x)) = x$, they are inverses!

ex 2) Find $f^{-1}(x)$. $f(x) = 2x+7$.

$y = 2x+7$ — Swap x & y

$x = 2y+7$ — Isolate y.

$\boxed{f^{-1}(x) = \frac{x-7}{2}} = y$

ex 3) Find $f^{-1}(x)$. $f(x) = 4x^3 - 1$

$y = 4x^3 - 1$; $x = 4y^3 - 1$; $\frac{x+1}{4} = y^3$;

$\boxed{f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}} = y$

ex 4) $f(x) = \frac{3}{x} - 1$. Find $f^{-1}(x)$.

$\frac{a}{b} = \frac{c}{d}$ swap

$x = \frac{3}{y} - 1$; $\frac{x+1}{1} = \frac{3}{y}$; $y = \boxed{\frac{3}{x+1} = f^{-1}(x)}$

ex 5) $f(x) = \frac{2x+1}{x}$. Find $f^{-1}(x)$.

$y = \frac{2x+1}{x}$; $x = \frac{2y+1}{y}$; $xy = 2y+1$; $xy - 2y = 1$; factor out a y

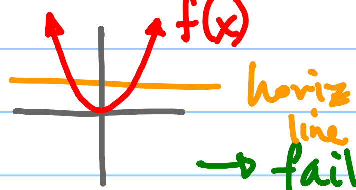
$y(x-2) = 1$;

$y = \boxed{\frac{1}{x-2} = f^{-1}(x)}$

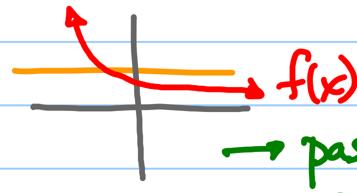
<u>Horizontal Line Test</u>

<u>notation</u>: does <u>not</u> guarantee if its a function <u>or</u> not.

→ checks if the $f^{-1}(x)$ is a function



→ fails...
$\therefore$ $f^{-1}(x)$ is <u>not</u> a function.



→ passes...
$\therefore$ $f^{-1}(x)$ is a function.

HW: p240, #2-34 even, & worksheet ("toad" side)