5/15 WED

Ch 10 - Sequences, Induction, & Probability

10.1 | Sequences & Summation Notation

ex) 1, 3, 5, 7 odd numbers

ex) 2, 4, 6, 8 even numbers

ex) 1, 1, 2, 3, 5, 8 Fibonacci sequence: add previous two numbers

ex) O, T, T, F, F, S, S, E "one, two, three..."

n : number of terms

a_n : n^{th} term (general term of a sequence)

a_1, a_2, a_3 ..... infinite sequence

10, 20, 30, ...

$a_1 = 10$ first term

3, 6, 9, 12 ← finite sequence

ex 1) $a_n = \frac{n+3}{2n-1}$. Find the first four terms of the sequence

$a_1 = \frac{1+3}{2(1)-1} = \frac{4}{1} = 4$; $a_2 = \frac{2+3}{2(2)-1} = \frac{5}{3}$; $a_3 = \frac{3+3}{2(3)-1} = \frac{6}{5}$

n

domain: $\{n = 1, 2, 3, 4\}$

$a_4 = \frac{4+3}{2(4)-1} = \frac{7}{7} = 1$ $\therefore$ $\boxed{4, \frac{5}{3}, \frac{6}{5}, 1}$

range

Recursion Formulas → requires previous terms ...

* position in a sequence:

$\ldots, a_7, a_8, a_9, a_{10}, \ldots$

YOU'RE HERE!

$\ldots, a_{n-2}, a_{n-1}, a_n, a_{n+1}, a_{n+2}, \ldots$

previous term

ex 2) $\underbrace{a_1=2}_{\text{clue}}$, $\underbrace{a_n=a_{n-1}-3}_{\text{recursive formula}}$, $n\geq 2$ domain. Find the 1st four terms.

previous term

$a_1 = 2$

$a_2 = a_{2-1} - 3$

n

$= a_1 - 3$

$a_2 = 2 - 3 = -1$

$a_3 = a_{3-1} - 3 = a_2 - 3 = -1 - 3 = -4$

$a_4 = a_{4-1} - 3 = a_3 - 3 = -4 - 3 = -7$ $\therefore$ $\boxed{2, -1, -4, -7}$

-3 -3 -3

ex 3) $a_1 = 3$, $a_n = 4\cdot a_{n-1} - 1$, $n \geq 2$. Find 1st four terms.

previous

$a_1 = 3$

$a_2 = 4(\quad) - 1 = 4\cdot 3 - 1 = 11$

$a_3 = 4(\quad) - 1 = 4\cdot 11 - 1 = 43$

$a_4 = 4(\quad) - 1 = 4\cdot 43 - 1 = 171$

$\boxed{3, 11, 43, 171}$

ex 4) $a_n = \frac{n^4}{(n-1)!}$. Find the 1st 4 terms.

factorial: ex) $4! = 4\cdot 3\cdot 2\cdot 1 = 24$

OPTN → F6 → PROB → $\boxed{x!}$

$0! = 1$

$a_1 = \frac{1^4}{(1-1)!} = \frac{1}{0!} = \frac{1}{1} = 1$;

$a_2 = \frac{2^4}{(2-1)!} = \frac{16}{1!} = \frac{16}{1} = 16$; $a_3 = \frac{3^4}{(3-1)!} = \frac{81}{2!} = \frac{81}{2}$; $a_4 = \frac{4^4}{(4-1)!} = \frac{256}{3!} = \frac{256}{6} = \frac{128}{3}$

ex 5) $\frac{9!}{7!2!} = \frac{9\cdot \cancel{8}^{4}\cdot\cancel{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}}{\cancel{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}\cdot\cancel{2\cdot1}} = \frac{36}{1} = 36$

ex 6) $\frac{5!}{7!} = \frac{\cancel{5\cdot4\cdot3\cdot2\cdot1}}{7\cdot6\cdot\cancel{5\cdot4\cdot3\cdot2\cdot1}} = \frac{1}{42}$

one less than (n+2), etc...

ex 7) $\frac{n(n+2)!}{(n+3)!} = \frac{n\,\cancel{(n+2)\cdot(n+1)(n)(n-1)\text{etc...}}}{(n+3)\cdot\cancel{(n+2)(n+1)(n)(n-1)\text{etc...}}} = \frac{n}{n+3}$

one less than (n+3), etc...

ex 8) $\frac{(n+4)!}{(n+2)!} = \frac{(n+4)(n+3)\cancel{(n+2)----}}{\cancel{(n+2)---}} = n^2+7n+12$

~~Summation Notion~~ → adding terms

$$\sum_{i=1}^{10}$$

10 ← end

Sigma

$i=1$: index (integer), start

ex 9) $\sum_{i=3}^{9} 9i = 9\cdot3 + 9\cdot4 + 9\cdot5 + 9\cdot6 + 9\cdot7 + 9\cdot8 + 9\cdot9$

$i=3$, $i=4$

$= 27+36+45+54+63+72+81$

$= 378$

ex 10) $\sum_{i=1}^{5}(i+11)$ or $\underbrace{\sum_{i=1}^{5} i + \sum_{i=1}^{5} 11}_{\text{two separate problems}}$

$= (1+11)+(2+11)+(3+11)+(4+11)+(5+11)$ or $1+2+3+4+5+5(11)$

$=70$

ex 11) $3+6+9+\ldots+15$. Express the sum using summation notation.

$3+6+9+12+15$

$$\sum_{i=1}^{5} 3i$$

$3+6+9+12+15$

1st term, 2nd term ... 5th term

Challenge *

ex 12) $2+8+18+\ldots+72$

* even #s: ÷2

$2\cdot1 \quad 2\cdot4 \quad 2\cdot9 \quad \ldots \quad 2\cdot36$ ← perfect squares

$1^2 \quad 2^2 \quad 3^2 \quad 6^2$

$$\sum_{i=1}^{6} 2\cdot i^2$$

ex 13) $5+6+7+8+\ldots+31$ * Multiple answers

-4 1st 2nd 3rd ... 27th

$$\sum_{i=1}^{27}(i+4) \quad \text{or} \quad \sum_{n=5}^{31} n$$

ex 14) $\overset{+3}{13+16} \overset{+3}{+19} \overset{+3}{+22} + \dots + 37$ pattern

25, 28, 31, 34

} 9 terms

$$\sum_{k=1}^{9} (\underbrace{3k}_{\text{multiples of 3}} + 10)$$

what plus 3 = 13?

Hw: p960
#2-60 even