10.2] Arithmetic Sequences

ex 1) 7, 11, 15, 19. Find the common difference: $d = a_n - a_{n-1}$ current - prev

$11 - 7 = +4$

ex 2) $-1, -4, -7, -10$. Common difference? $-4 - -1 = -3$

$a_1 \to 1 = 1$ a_1 — common difference

$a_2 \to 4 = 1 + 3$

$a_3 \to 7 = 1 + 3 + 3$

$a_4 \to 10 = 1 + 3 + 3 + 3$

General Term of an Arithmetic Sequence

$$a_n = a_1 + (n-1)d$$

ex 3) Find the 1st 5 terms... & the general formula.

a) $a_1 = 3$, $d = 4$

$\to 3, 7, 11, 15, 19$

$+4 \leftarrow d$

$a_n = a_1 + (n-1)d$
$a_n = 3 + (n-1)4$
$a_n = 3 + 4n - 4$
$a_n = 4n - 1$

b) $a_1 = -15$, $d = 3$

$\to -15, -12, -9, -6, -3$

$+3 \leftarrow d$

$a_n = a_1 + (n-1)d$
$a_n = -15 + (n-1)3$
$a_n = -15 + 3n - 3$
$a_n = 3n - 18$

ex 4) Find the indicated term

a) a_{28}: $a_1 = 1$, $d = 11$ $\to$ $a_n = a_1 + (n-1)d$

$a_{28} = 1 + (28 - 1) \cdot 11 = 298$

b) a_{13}: $a_1 = 29$, $d = -3$ $\to$ $a_n = a_1 + (n-1)d$

$a_{13} = 29 + (3 - 1) \cdot -3 = -7$

ex 5) General formula & the 20th term.

a) $1, 5, 9, 13, ...$ ← $d = 5 - 1 = 4$

$a_n = a_1 + (n-1)d$

$a_n = 1 + (n-1)4$

$a_n = 1 + 4n - 4$

$\boxed{a_n = 4n - 3}$ OR

$a_{20} = 1 + (20-1)4$

$a_{20} = \boxed{77}$

$a_{20} = 4(20) - 3 = \boxed{77}$

b) $a_n = \underset{\text{previous}}{a_{n-1}} + 3, \quad a_1 = 4$

recursion formula

$\rightarrow a_1 = \boxed{4}$ d

$a_2 = (4) + 3 = \boxed{7}$

$a_3 = \boxed{10}, \ a_4 = \boxed{13}, ...$

$a_n = a_1 + (n-1)d$

$a_n = 4 + (n-1)3$

$a_n = 4 + 3n - 3$

$\boxed{a_n = 3n + 1}$

$a_{20} = 3(20) + 1 = \boxed{61}$

Sum of the 1st n terms:

$$S_n = \frac{n}{2}(a_1 + a_n)$$ ← Why?

2 pairs

$\underbrace{4 + 5 + 6 + 7}_{\text{4 terms}} = 2 \cdot 11$

$\frac{4}{2}(4 + 7)$

1st   last

ex 6) Sum of 1st 30 terms
given $d = -5$, $a_1 = 10$.

$\rightarrow 10, 5, 0, -5, \text{etc...}$

$S_n = \frac{n}{2}(a_1 + a_n)$?

$S_{30} = \frac{30}{2}(10 + \quad)$

$S_{30} = 15(10 + -135) = \boxed{-1875}$

$a_n = a_1 + (n-1)d$

$a_{30} = 10 + (30-1) \cdot -5$

$a_{30} = \boxed{-135}$

ex 7) Sum of 1st 40 terms:
$-19, -23, -27, ...$ $d = \boxed{-4}$

$S_{40} = \frac{40}{2}(-19 + \boxed{a_{40}})$?

$a_n = a_1 + (n-1)d$

$a_{40} = -19 + (40-1) \cdot -4 = \boxed{-175}$

$S_{40} = 20(-19 + -175)$

$= \boxed{-3880}$

ex 8) $\displaystyle\sum_{i=1}^{45}(-2i - 6)$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{45} = \frac{45}{2}(a_1 + a_{45})$

$-2(1) - 6 = \boxed{-8}$ ← Subt 1 for i

$-2(45) - 6 = \boxed{-96}$ ← Subt 45 for i

$S_{45} = \frac{45}{2}(-8 + -96) = \boxed{-2340}$

HW: p 969, # 2-50 even