5/24 TUE

10.3 ..... Annuities (part 1)

$$A = P(1+r)^{t}$$

$$A = P\left(1+\frac{r}{n}\right)^{n \cdot t}$$

Compound Interest

A: final amount; P: initial amount; r: interest rate; n: # times interest is compounded per year; t: time

ex 1) \$5000 → bank account : 3.25% APR (annual percentage rate), 6 years, compounded quarterly. How much will you have at the end of 6 years?

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 5000\left(1+\frac{.0325}{4}\right)^{4 \cdot 6} = \$6071.77$$

$$-5000.00$$

How much interest was earned? ?

\$1071.77

ex 2) See #1 .... interest is compounded continuously.

$$A = Pe^{rt} = 5000e^{.0325(6)} = \$6076.55$$

e: natural base

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Annuities - sequence of equal payments made at equal time periods. ex) mortgage, car payments, bank account

Present Annuity

$$P_n = P\left[\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}\right]$$

P_n: Present Value; P: Payment

ex) Buy a house or a car NOW

Future Annuity

$$F_n = P\left[\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}\right]$$

F_n: Future Value

ex) Save up to buy something in the future or I want a certain \$\$\$ for retirement ex 3) Dylan can afford \$200 per month for a car: 5 yr loan @ 5.5% APR. How much can the car cost? → Present (Now)

$$P_n = P\left[\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\left(\frac{r}{n}\right)}\right] \qquad P_n = 200\left[\frac{\left(1-\left(1+\frac{.055}{12}\right)^{(-12\cdot 5)}\right)}{\left(\frac{.055}{12}\right)}\right]$$

$= \$10470.57$

How much does Dylan pay?

→ \$200 × 12 × 5 = \$12000

ex 4) Justin buys a house \$400,000. ← Present Annuity

4.5% for 30 years. What is his monthly payments?

$$P_n = P\left[\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\left(\frac{r}{n}\right)}\right] \qquad 400{,}000 = P\left[\frac{1-\left(1+\frac{.045}{12}\right)^{-12(30)}}{\left(\frac{.045}{12}\right)}\right]$$

$\div$

$P = \$2026.74$

How much does Justin pay out over the course of the loan? → \$2026.74 × 12 × 30 = \$729,626.40