5/20 MON

10.3 Geometric Sequences & Series

Common Ratio: $r = \frac{\text{current term}}{\text{previous term}}$ → $r = \frac{a_n}{a_{n-1}}$

ex 1) Geometric finite sequence: $3, 6, 12, 24, 48$ $\quad r = \frac{6}{3} = 2$

ex 2) $\frac{3}{3}, \frac{3}{12}, \frac{3}{48}, \frac{3}{192}, \frac{3}{768}, \ldots$ geom ∞ seq. $\quad r = \frac{\frac{3}{12}}{\frac{3}{3}} = \frac{1}{4}$

ex 3) $a_1 = 5$, $r = 4$. Find 1st 5 terms. $\quad r = \frac{3}{\frac{12}{3}} = \frac{1}{4}$

$5, \overset{\times 4}{20}, \overset{\times 4}{80}, \overset{\times 4}{320}, \overset{\times 4}{1280}$

ex 4) $a_1 = 6$, $r = \frac{1}{3}$. Find 1st 5 terms.

$6, \overset{\times \frac{1}{3}}{2}, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}$

General Term of a Geometric Sequence

$$a_n = a_1 \cdot r^{n-1}$$

ex 5) Find a_6 given $a_1 = 5$, $r = -2$

$a_n = a_1 \cdot r^{n-1} \rightarrow a_6 = 5 \cdot (-2)^{6-1} \rightarrow a_6 = 5(-2)^5 \rightarrow -160$

ex 6) Write the general formula for a_n:

$3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \frac{3}{16}, \ldots$

$r = \frac{-\frac{3}{2}}{3} \rightarrow -\frac{3}{2} \div 3 \rightarrow -\frac{3}{2} \cdot \frac{1}{3} = -\frac{1}{2}$

$a_n = a_1 \cdot r^{n-1}$?

$$a_n = 3 \cdot \left(-\frac{1}{2}\right)^{n-1}$$

Sum of Geometric Sequence

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

ex 7) Find S_{13}. Given $7, -14, 28, -56, \ldots$

$\frac{-14}{7} \rightarrow r = -2$

$$S_{13} = \frac{7(1 - (-2)^{13})}{\underbrace{1 - (-2)}_{3}} = 19{,}117$$

ex 8) $\sum_{i=1}^{8}\left(\frac{3}{5}\right)^i$ geometric S_n? $S_n = \frac{a_1(1-r^n)}{1-r}$ plug in 1 for i! $\left(\frac{3}{5}\right)^1$

$S_8 = \frac{\frac{3}{5}\left(1-\left(\frac{3}{5}\right)^8\right)}{1-\frac{3}{5}} = 1.475$

Sum of an Infinite Geometric Series (Sum of Sequence Terms)

$S = \frac{a_1}{1-r}$ $|r|<1$ (otherwise Sum DNE)

ex 9) $100 + 50 + 25 + 12.5 + \ldots$

$r = \frac{1}{2}$: geometric $S = \frac{a_1}{1-r} = \frac{100}{1-\frac{1}{2}} = \frac{100}{\frac{1}{2}} = 100 \div \frac{1}{2} = 100 \cdot 2 = 200$

ex 10) $\frac{1}{3} - 1 + 3 - \ldots$

$r = \frac{-1}{\frac{1}{3}} = -1 \cdot \frac{3}{1} = -3$ Sum DNE

ex 11) Express the repeating decimal (pattern) as a fraction !!!

a) $.7 + .07 + .007 + .0007 + \ldots$

$r = \frac{.07}{.7} = \frac{1}{10}$ $S = \frac{a_1}{1-r} = \frac{\frac{7}{10}}{1-\frac{1}{10}} = \frac{\frac{7}{10}}{\frac{9}{10}} = \frac{7}{10} \div \frac{9}{10} = \frac{7}{10} \cdot \frac{10}{9} = \frac{7}{9}$ ($a_1 = .7 = \frac{7}{10}$)

b) $.\overline{972}$

$= .972972972\ldots$

$= .972 + .000972 + .000000972 + \ldots$ ($.972 = \frac{972}{1000}$)

$r = \frac{1}{1000}$

$S = \frac{\frac{972}{1000}}{1-\frac{1}{1000}} = \frac{\frac{972}{1000}}{\frac{999}{1000}} = \frac{972}{1000} \div \frac{999}{1000} = \frac{972}{1000} \cdot \frac{1000}{999} = \frac{972}{999} = \frac{36}{37}$

ex 12) Identify the sequence.

a) $a_n = n+5$ arithmetic, d

b) $a_n = 2^n$ geometric, r

c) $a_n = n^2+5$ neither (quadratic)

hw: p983, #2-56 even