Exercise Set 11.3

Practice Exercises

In Exercises 1–18, use the definition of continuity to determine whether f is continuous at a.

1. $f(x) = 2x + 5$, $a = 1$

2. $f(x) = 3x + 4$, $a = 1$

3. $f(x) = x^2 - 3x + 7$, $a = 4$

4. $f(x) = x^2 - 5x + 6$, $a = 4$

5. $f(x) = \dfrac{x^2 + 4}{x - 2}$, $a = 3$

6. $f(x) = \dfrac{x^2 + 6}{x - 5}$, $a = 6$

7. $f(x) = \dfrac{x + 5}{x - 5}$, $a = 5$

8. $f(x) = \dfrac{x + 7}{x - 7}$, $a = 7$

9. $f(x) = \dfrac{x - 5}{x + 5}$, $a = 5$

10. $f(x) = \dfrac{x - 7}{x + 7}$, $a = 7$

11. $f(x) = \dfrac{x^2 + 5x}{x^2 - 5x}$, $a = 0$

12. $f(x) = \dfrac{x^2 + 8x}{x^2 - 8x}$, $a = 0$

13. $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$, $a = 2$

14. $f(x) = \begin{cases} \dfrac{x^2 - 36}{x - 6} & \text{if } x \neq 6 \\ 13 & \text{if } x = 6 \end{cases}$, $a = 6$

15. $f(x) = \begin{cases} x - 5 & \text{if } x \leq 0 \\ x^2 + x - 5 & \text{if } x > 0 \end{cases}$, $a = 0$

16. $f(x) = \begin{cases} x - 4 & \text{if } x \leq 0 \\ x^2 + x - 4 & \text{if } x > 0 \end{cases}$, $a = 0$

17. $f(x) = \begin{cases} 1 - x & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ x^2 - 1 & \text{if } x > 1 \end{cases}$, $a = 1$

18. $f(x) = \begin{cases} 2 - x & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$, $a = 1$

In Exercises 19–34, determine for what numbers, if any, the given function is discontinuous.

19. $f(x) = x^2 + 4x - 6$

20. $f(x) = x^2 + 8x - 10$

21. $f(x) = \dfrac{x + 1}{(x + 1)(x - 4)}$

22. $f(x) = \dfrac{x + 2}{(x + 2)(x - 5)}$

23. $f(x) = \dfrac{\sin x}{x}$

24. $f(x) = \dfrac{1 - \cos x}{x}$

25. $f(x) = \pi$

26. $f(x) = c$

27. $f(x) = \begin{cases} x - 1 & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$

28. $f(x) = \begin{cases} x - 2 & \text{if } x \leq 2 \\ x^2 - 1 & \text{if } x > 2 \end{cases}$

29. $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$

30. $f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

31. $f(x) = \begin{cases} x + 6 & \text{if } x \leq 0 \\ 6 & \text{if } 0 < x \leq 2 \\ x^2 + 1 & \text{if } x > 2 \end{cases}$

32. $f(x) = \begin{cases} x + 7 & \text{if } x \leq 0 \\ 7 & \text{if } 0 < x \leq 3 \\ x^2 - 1 & \text{if } x > 3 \end{cases}$

33. $f(x) = \begin{cases} 5x & \text{if } x < 4 \\ 21 & \text{if } x = 4 \\ x^2 + 4 & \text{if } x > 4 \end{cases}$

34. $f(x) = \begin{cases} 7x & \text{if } x < 6 \\ 41 & \text{if } x = 6 \\ x^2 + 6 & \text{if } x > 6 \end{cases}$

Practice Plus

In Exercises 35–38, graph each function. Then determine for what numbers, if any, the function is discontinuous.

35. $f(x) = \begin{cases} \sin x & \text{if } -\pi \leq x < 0 \\ -\sin x & \text{if } 0 \leq x < \pi \\ \cos x & \text{if } \pi \leq x \leq 2\pi \end{cases}$

36. $f(x) = \begin{cases} -\cos x & \text{if } -\pi \leq x < 0 \\ -\sin x & \text{if } 0 \leq x < \pi \\ \sin x & \text{if } \pi \leq x \leq 2\pi \end{cases}$

37. $f(x) = \begin{cases} -1 & \text{if } x \text{ is an integer.} \\ 1 & \text{if } x \text{ is not an integer.} \end{cases}$

38. $f(x) = \begin{cases} 2 & \text{if } x \text{ is an odd integer.} \\ -2 & \text{if } x \text{ is not an odd integer.} \end{cases}$

In Exercises 39–42, determine for what numbers, if any, the function is discontinuous. Construct a table to find any required limits.

39. $f(x) = \begin{cases} \dfrac{\sin 2x}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$

40. $f(x) = \begin{cases} \dfrac{\sin 3x}{x} & \text{if } x \neq 0 \\ 3 & \text{if } x = 0 \end{cases}$

41. $f(x) = \begin{cases} \dfrac{\cos x}{x - \dfrac{\pi}{2}} & \text{if } x \neq \dfrac{\pi}{2} \\ 1 & \text{if } x = \dfrac{\pi}{2} \end{cases}$

42. $f(x) = \begin{cases} \dfrac{\sin x}{x - \pi} & \text{if } x \neq \pi \\ 1 & \text{if } x = \pi \end{cases}$