2/19 TUE

2.5 (part 1) | Zeros of Polynomial Functions

* Use Graphing Calculator to find the 1st root.....

ex 1) Find all zeros of $f(x) = x^3 + 2x^2 - 5x - 6$.

GC → possible roots: -3, -1, 2 → Test w/ Synthetic ÷

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -5 & -6 \\ & & -3 & 3 & 6 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array}$$

+ ×

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x - 2 = 0 \quad x + 1 = 0$

$x = 2 \quad x = -1$

Find the rest of the zeros (ac, guess, quad formula)

∴ zeros: -3, -1, 2

ex 2) Find zeros of $f(x) = x^3 - 8x^2 + 16x - 8$

GC → possible roots: 2, blah...

$$\begin{array}{r|rrrr} 2 & 1 & -8 & 16 & -8 \\ & & 2 & -12 & 8 \\ \hline & 1 & -6 & 4 & \boxed{0} \end{array}$$

$x^2 - 6x + 4 = 0$:(

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 - 16}}{2} = \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2}$$

$$= \frac{\cancel{2}(3 \pm 1\sqrt{5})}{\cancel{2}}$$

∴ zeros: 2, $3+\sqrt{5}$, $3-\sqrt{5}$

ex 3) Zeros? $f(x) = x^3 + 8x^2 + 25x + 26$

GC → possible roots: -2, blah (imag?)...

$$\begin{array}{r|rrrr} -2 & 1 & 8 & 25 & 26 \\ & & -2 & -12 & -26 \\ \hline & 1 & 6 & 13 & \boxed{0} \end{array}$$

$x^2 + 6x + 13 = 0$:(

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(13)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 52}}{2}$$

$$= \frac{-6 \pm \sqrt{-16}}{2} = \frac{-6 \pm 4i}{2} = \frac{\cancel{2}(-3 \pm 2i)}{\cancel{2}}$$

zeros: $\{-2, -3+2i, -3-2i\}$

Linear Factorization Theorem

→ Build a polynomial from roots ... & function value → stretch factor

$$f(x) = a_n (x - c_1)(x - c_2) \ldots (x - c_n)$$

$c_1, c_2, \ldots, c_n$: Complex numbers

a_n: stretch factor

\# of complex roots

clue: stretch factor

ex 4) $n=3$. 3 and i are zeros. $f(2)=25$

1st root, 2nd root, 3rd root?

$0+1i \rightarrow 0-1i$

complex conjugate of 2nd root → $a+bi$ & $a-bi$

$f(x)=a_3\cdot(x-3)\underbrace{(x-i)(x--i)}_{x^2-i^2}$ FOIL

$i^2 = -1$

$f(x)=a_3(x-3)(x^2+1)$ a_3?

$f(2)=25=a_3(2-3)(2^2+1)$

$25=a_3(-1)(5)$

$25=-5a_3$

$-5=a_3$

$f(x)=-5(x-3)(x^2+1)$

$=-5(x^3+x-3x^2-3)$

$\boxed{f(x)=-5x^3+15x^2-5x+15}$

ex 5) $n=4$. zeros: $3, \frac{1}{3}, 1+2i$. $f(1)=48$

conjugate: $1-2i$

$f(x)=a_4(x-3)(x-\frac{1}{3})(x-(1+2i))(x-(1-2i))$

$f(x)=a_4\underbrace{(x-3)(3x-1)}_{3x^2-x-9x+3}\underbrace{(x-1-2i)(x-1+2i)}_{x^2-x+\cancel{2ix}-x+1-\cancel{2i}-\cancel{2ix}+\cancel{2i}-4i^2}$ FOIL

$i^2=-1$

$f(x)=a_4(3x^2-10x+3)(x^2-2x+5)$

$f(1)=48=a_4(3(1)^2-10(1)+3)((1)^2-2(1)+5)$

$48=a_4(-4)(4)$

$48=-16a_4$

$-3=a_4$

$3x^4-6x^3+15x^2$

$-10x^3+20x^2-50x$

$+3x^2-6x+15$

$f(x)=-3(3x^4-16x^3+38x^2-56x+15)$

$\boxed{f(x)=-9x^4+48x^3-114x^2+168x-45}$

HW p336, #21-24, 26, 32