2/21 THU

Ch 2.5 (part 2) | Zeros of Polynomial Functions

Rational Zero Theorem

→ factors of the constant & leading coefficient

ex 1) $1x^5 - 4x^2 + 6x + 5$ — List all possible roots.

leading coefficient $q: \pm 1$

constant $p: \pm 1, \pm 5$

possible roots $\frac{p}{q}: \underbrace{\pm 1, \pm 5}_{\text{4 possible roots}}$

ex 2) List all possible roots of $f(x) = -4x^4 + 4x^2 - 2x + 6$

q = -4, p = 6

$p: \pm 1, \pm 2, \pm 3, \pm 6$

$q: \pm 1, \pm 2, \pm 4$

$\frac{p}{q}: \underbrace{\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm 6}_{\text{16 possible roots}}$

Decartes Rule of Signs

→ # of positive or negative real roots

Positives: # sign changes of $f(x)$

negatives: # sign changes of $f(-x)$

ex 3) Determine # of pos or neg real roots

$f(x) = -5x^7 + x^4 + x^2 + x + 9$

positives? yes no no no → 1 positive zero (root)

$f(-x) = -5(-x)^7 + (-x)^4 + (-x)^2 + (-x) + 9$

$5x^7 + x^4 + x^2 - x + 9$

negatives? no no yes yes → 2 or 0 negative roots

subtract 2

ex 4) # roots? $f(x) = x^7 + x^4 + x^2 + x + 9$

pos? no sign changes → 0 positive roots

$f(-x) = (-x)^7 + (-x)^4 + (-x)^2 + (-x) + 9$

$= -x^7 + x^4 + x^2 - x + 9$ → 3 or 1 neg roots

neg?

ex 5) Find all roots of $f(x) = 1x^3 + 2x^2 - 5x - 6$

$p: \pm 1, \pm 2, \pm 3, \pm 6$; $q: \pm 1$; $\frac{p}{q}: \underbrace{\pm 1, \pm 2, \pm 3, \pm 6}_{8 \text{ possible roots}}$

$f(x) = x^3 + 2x^2 - 5x - 6 \rightarrow$ 1 positive root

$f(-x) = (-x)^3 + 2(-x)^2 - 5(-x) - 6$

$-x^3 + 2x^2 + 5x - 6 \rightarrow$ 2 or 0 negative roots

+	×
↓	↘

$$\begin{array}{c|cccc}
+ & 1 & 2 & -5 & -6 \\
 & & \cancel{1} & \cancel{3} & \cancel{-2} \\
\hline
 & \cancel{1} & \cancel{3} & \cancel{-2} & \cancel{-4} \\
2 & & 2 & 8 & 6 \\
\hline
 & 1 & 4 & 3 & \boxed{0}
\end{array}$$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -3 \mid x = -1$

$\therefore$ roots: $\{2, -3, -1\}$

ex 6) $f(x) = x^3 + 7x^2 + 19x + 13$

$p: \pm 1, \pm 13$; $q: \pm 1$; $\frac{p}{q}: \pm 1, \pm 13$

$f(x) =$ no sign changes $\rightarrow$ 0 pos roots

$f(-x) = (-x)^3 + 7(-x)^2 + 19(-x) + 13$

$= -x^3 + 7x^2 - 19x + 13 \rightarrow$ 3 or 1 neg roots

$$\begin{array}{c|cccc}
-1 & 1 & 7 & 19 & 13 \\
 & & -1 & -6 & -13 \\
\hline
 & 1 & 6 & 13 & \boxed{0}
\end{array}$$

$x^2 + 6x + 13 = 0$

$x = \frac{-(6) \pm \sqrt{6^2 - 4(1)(13)}}{2(1)}$

$= \frac{-6 \pm \sqrt{36 - 52}}{2}$

$= \frac{-6 \pm \sqrt{-16}}{2} = \frac{-6 \pm 4i}{2}$

$= -3 \pm 2i$

$\therefore \{-1, -3+2i, -3-2i\}$

HW: p336, # 17–20, 39, 41