2/22 FRI

2.6 (part 1) Rational Functions & Their Graphs

→ $f(x) = \dfrac{p(x)}{q(x)}$ ← $q(x) \neq 0$ (restriction on the domain)

Find the domain

ex 1) $h(x) = \dfrac{8x}{x-9}$

$x - 9 \neq 0$

$x - 9 \neq 0$

$x \neq 9$ → d: $\{x \mid x \neq 9\}$ or $(-\infty, 9) \cup (9, \infty)$

such that

set builder notation

interval notation

ex 2) $h(x) = \dfrac{x+8}{x^2-4}$

$x^2 - 4 \neq 0$

$(x-2)(x+2) \neq 0$ → d: $\{x \mid x \neq \pm 2\}$

$x \neq 2 \mid x \neq -2$

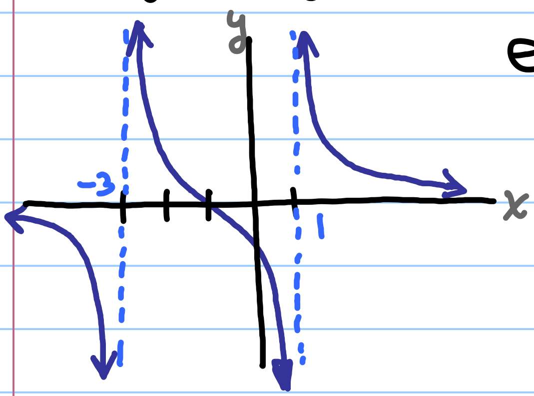
ex 3) $h(x) = \dfrac{x+9}{x^2+16}$

$x^2 + 16 \neq 0$ → d: $\{x \mid \mathbb{R}\}$ ← all real #s

$x^2 \neq -16$

Arrow Notation

Symbols	Meaning
$x \to a^+$	x approaches a from the right
$x \to a^-$	" " " " " left
$x \to \infty$	" " Infinity (to the right)
$x \to -\infty$	" " negative infinity (to the left)



ex 4)

a) $x \to -3^-$, $f(x) \to \underline{-\infty}$ ← going down

b) $x \to -3^+$, $f(x) \to \underline{\infty}$

c) $x \to 1^-$, $f(x) \to \underline{-\infty}$

d) $x \to 1^+$, $f(x) \to \underline{\infty}$

e) $x \to -\infty$, $f(x) \to \underline{0}$

f) $x \to \infty$, $f(x) \to \underline{0}$

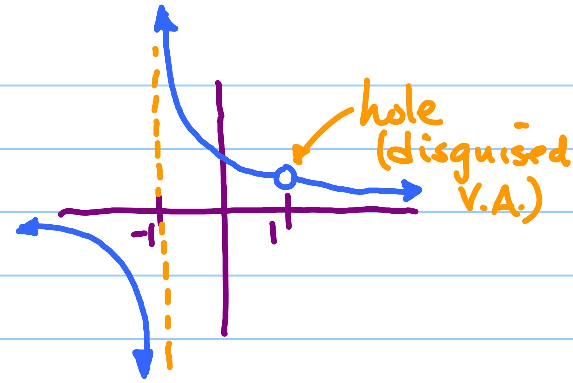
Vertical Asymptotes / Holes

"V.A." "Holes"

restrictions on the domain — canceled factors

$$f(x)=\frac{x-1}{x^2-1}=\frac{\cancel{x-1}}{\cancel{(x-1)}\underbrace{(x+1)}_{\text{V.A.}}}$$

hole



ex 5) $f(x)=\frac{x}{x+5}$. Find any vert. asymptotes and/or holes

V.A.? set denom $=0$

$x+5=0$

$x=-5$

Holes

None

ex 6) V.A/holes? $f(x)=\frac{\cancel{x-4}}{x\cancel{(x-4)}}$.

VA.

$x=0$

~~$x=4$~~

Holes

$x-4=0$

$x=4$

ex 7) $f(x)=\frac{(x^2-81)}{(x^2-6x-27)}=\frac{(x+9)\cancel{(x-9)}}{\cancel{(x-9)}(x+3)}$

Holes

$x-9=0$

$x=9$

V.A.

$x+3=0$

$x=-3$

Horizontal Asymptotes

"H.A."

→ semi-permeable "force fields"

→ $y=\ldots$

$f(x)=\frac{p(x)}{q(x)}$: If degree of $p(x)$ $<$ degree of $q(x)$,

then there is a H.A.: $y=0$

If " " $p(x)$ $=$ " " $q(x)$,

then H.A.: $y=\frac{a}{b}$

a & b are leading coefficients of $p(x)$ & $q(x)$ respectively.

If " " $p(x)$ $>$ " " $q(x)$

then there are <u>no</u> H.A.

ex 8) $f(x) = \dfrac{8x}{2x^2+1}$ $\leftarrow$ degree 1, $\leftarrow$ degree 2 $\quad 1 < 2 \therefore$ H.A. $y=0$

H.A.?

ex 9) $f(x) = \dfrac{10x^2}{2x^2+1}$ $\leftarrow$ deg 2, $\leftarrow$ deg 2 $\quad 2 = 2 \therefore$ H.A. $y = \frac{a}{b} = \frac{10}{2} = 5$

H.A.?

ex 10) $f(x) = \dfrac{10x^3}{5x^2+1}$ $\leftarrow$ deg 3, $\leftarrow$ deg 2 $\quad 3 > 2 \therefore$ H.A. none

H.A.?

Hw. p 354, #1-8, 15-20, 22-36

#22-28 (add holes)