3/12 TUE

3.2 Logarithmic Functions

Common base → $\log$ (10^x) base 10 | $\ln$ (e^x) base e ← natural base

$2^x = 4 \rightarrow x = 2$

$2^x = 8 \rightarrow x = 3$

$2^x = 5 \rightarrow ?$ (solve for exponent)

x = somewhere between 2 & 3. (closer to 2)

exponential form ⇨ logarithmic form

$2^x = 5$ (power: x; base: 2; result: 5)

$\log_2 5 = x$ ← "log base 2 of 5 equals x"

$b^y = x$ $\qquad$ $\log_b x = y$

ex 1) $\log_4 16 = 2$ → exponential form

$4^2 = 16$

ex 2) $\log_6 216 = x$. Find x.

→ convert to exponential form: $6^x = 216$

$216 = 6 \cdot 36 = 6 \cdot 6 \cdot 6$

convert to the same base: $6^x = 6^3$

$x = 3$

ex 3) Change to log form. $\sqrt[3]{343} = 7$ →

$343^{\frac{1}{3}} = 7$

$\log_{343} 7 = \frac{1}{3}$

ex 4) Change to log form: $e^y = 33$ →

$\log_e 33 = y$ (aka $\ln 33 = y$)

ex 5) Use a calculator to find:

a) $\log 57 \approx 1.7559$ b) $\ln 33 \approx 3.4965$

$\log_{10} 57 = x$ ... $10^{1.7559} = 57$

ex 6) Evaluate w/out a calculator.

a) $\log_2 4 = x$ → convert to exp form: $2^x = 4$, $x = 2$

b) $\log_3 \sqrt{3} = x$, $3^x = \sqrt{3}$, $3^x = 3^{\frac{1}{2}}$, $x = \frac{1}{2}$

c) $\log_5 \frac{1}{\sqrt{5}}$ $\log_5 \frac{1}{\sqrt{5}} = x$, $5^x = \frac{1}{\sqrt{5}}$, $5^x = 5^{-\frac{1}{2}}$, $x = -\frac{1}{2}$

Basic Properties of Logs

$\log_b b = 1$ ($b^x = b^1$) | $\log_b 1 = 0$ ($b^x = 1$)

Inverse Properties

$\log_b b^x = x$

$b^{\log_b x} = x$

ex 7) $\log_9 9^1$ ($9^x = 9^1$) $= 1$

ex 8) $\log_8 1$ ($8^x = 1$) $= 0$

ex 9) $6^{\log_6 15} = 15$

ex 10) $\log_7 7^8 = 8$

HW p410 #2-42 even 82-100 even

ex 11) $\log .0001$ → base 10 → same base

$\log_{10} \frac{1}{10000} \rightarrow \log_{10} \frac{1}{10^4} \rightarrow \log_{10} 10^{-4}$

$= -4$

ex 12) $\ln \sqrt[4]{e} \rightarrow \ln e^{\frac{1}{4}} \rightarrow \boxed{\frac{1}{4}}$

aka $\log_e \sqrt[4]{e} \rightarrow e^x = \sqrt[4]{e} \rightarrow e^x = e^{\frac{1}{4}}$

ex 13) $\ln e^{6x} \rightarrow \boxed{6x}$