3/14 THU

3.3 Properties of Logs

"one log" "two logs"

Product: $\log_b m \cdot n = \log_b m + \log_b n$ ← $a^m \cdot a^n = a^{m+n}$

Quotient: $\log_b \frac{m}{n} = \log_b m - \log_b n$ ← $\frac{a^m}{a^n} = a^{m-n}$

Power: $\log_b m^p = p \cdot \log_b m$

~~$\log(m+n) = \log m \cdot \log n$~~

~~$\log(m-n) = \frac{\log m}{\log n}$~~

ex) $2^x = 314$ (exponential form) → $\log_2 314 = x$ (log form; 2 = base)

* Change the Base! : base 2 → base 10 (GC)

Change of Base: $\log_b m = \frac{\log_{10} m}{\log_{10} b}$ base $b \to 10$

$\log_2 314 = \frac{\log 314}{\log 2}$ → GC → $x = 8.295$

ex 1) Expand the logs (use properties)

a) $\log_6 77$ → $\log_6(7 \cdot 11) = \log_6 7 + \log_6 11$

b) $\log(100x)$ (base 10) → $\log(10 \cdot 10 \cdot x)$ → $\log 10 + \log 10 + \log x$ → $1 + 1 + \log x = 2 + \log x$

$\log(10^2 \cdot x)$ → $\log 10^2 + \log x$

c) $\log_3\left(\frac{23}{x}\right) = \log_3 23 - \log_3 x$

d) $\ln\left(\frac{e^5}{11}\right) = \ln e^5 - \ln 11 = 5 - \ln 11$

e) $\log_6 3^9 = 9 \cdot \log_6 3$

PE
MD
AS

f) $\ln \sqrt[3]{x} = \ln x^{\frac{1}{3}} = \boxed{\frac{1}{3}\ln x}$

g) $\log (x+4)^2 = \log (x+4)^{(2)} = \boxed{2\log (x+4)}$

h) $\log_b (x^4 \sqrt[3]{y}) = \log_b x^4 + \log_b y^{\frac{1}{3}}$

$= \boxed{4\log_b x + \frac{1}{3}\log_b y}$

i) $\log_5 \left(\frac{\sqrt{x}}{25y^3}\right) = \log_5 x^{\frac{1}{2}} - \log_5 25y^3$

$= \frac{1}{2}\log_5 x - (\log_5 25 + \log_5 y^3)$

$= \frac{1}{2}\log_5 x - \log_5 25 - 3\log_5 y$ ($25 = 5^2$)

$= \frac{1}{2}\log_5 x - \log_5 5^2 - 3\log_5 y$

$= \boxed{\frac{1}{2}\log_5 x - 2 - 3\log_5 y}$

ex 2) Condense the logs (→ single log)

a) $\log 25 + \log 4 = \log(25 \cdot 4) = \log 100 = \log_{10} 10^2 = \boxed{2}$

b) $\log(7x+6) - \log x = \boxed{\log \frac{7x+6}{x}}$

c) $2\ln x + \frac{1}{3}\ln(x+5) = \ln x^2 + \ln(x+5)^{\frac{1}{3}}$

power prop. product prop.

$\boxed{\ln\left[x^2(x+5)^{\frac{1}{3}}\right]}$

d) $2\log(x-3) - \log x$

$\log(x-3)^2 - \log x = \boxed{\log \frac{(x-3)^2}{x}}$

e) $\frac{1}{4}\log_b x - 2\log_b 5 - 10\log_b y$

$\log_b x^{\frac{1}{4}} - \log_b 5^2 - \log_b y^{10}$

$\log_b \frac{x^{\frac{1}{4}}}{5^2 y^{10}} = \log_b \frac{\sqrt[4]{x}}{25y^{10}}$

ex 3) use change of base formula: $\log_7 2506$

base 7 → base 10

$\frac{\log 2506}{\log 7} \approx 4.02$

$7^x = 2506$

$7^{4.02} \approx 2506$

Hw: p421, # 2-78 even