3/22 FRI

3.5 Exponential Growth & Decay; Modeling Data

$y = a \cdot b^x$

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

BIG

Continuously Compounded

$$A = Pe^{rt}$$

$$A = A_0 e^{kt}$$

A: final; A_0: initial; k: growth rate

ex 1) Investment: 2000

$$A = 3100e^{0.046t}$$

3100: initial investment; 0.046: growth rate; t: ?

When will the account be worth $3902?

$$\frac{3902}{3100} = \frac{3100e^{0.046t}}{3100}$$

$$\frac{3902}{3100} = e^{0.046t}$$

*ln both sides

$$\ln\left(\frac{3902}{3100}\right) = \ln e^{0.046t}$$

$$\frac{\ln\left(\frac{3902}{3100}\right)}{0.046} = t \approx 5 \text{ year}$$

$5 + 2000 = 2005$

ex 2) Popl (1984): 22 million / Popl (1994): 31 million

$$A = 22e^{kt}$$

$$31 = 22e^{k(10)}$$

$$\frac{31}{22} = e^{10k}$$

$$\ln\left(\frac{31}{22}\right) = 10k$$

$$\frac{\ln\left(\frac{31}{22}\right)}{10} = k \approx 0.0343$$

ex 3) 

$$A = A_0 e^{-0.00693x}$$

Radioactive Material: 900 lbs

How much is left after 30yrs?

$$A = 900e^{-.00693(30)}$$

$$\approx 731.1 \text{ lbs.}$$

ex 4) Half-life of silicon-32: 710 years.

50 g → How much will be present in 200 years?

$A = A_0 e^{kt}$

$\frac{25}{50} = \frac{50}{50} e^{k(710)}$

$\frac{1}{2} = e^{710k}$ * ln both sides

$\ln \frac{1}{2} = \ln e^{710k}$

Find k.

$\ln \frac{1}{2} = 710k$

$\frac{\ln \frac{1}{2}}{710} = k$

$A = 50 e^{\left(\frac{\ln \frac{1}{2}}{710}\right)(200)} \approx 41.131 \text{ g}$

ex 5) Leaf: 15% carbon 14. Half-life carbon-14: 5600 years

How old is the fossil?

$A = A_0 e^{kt}$

$\frac{A}{A_0} = \frac{1}{2}$

$\frac{1}{2} = e^{k(5600)}$

$\ln \frac{1}{2} = \ln e^{5600k}$ * Find k.

$\ln \frac{1}{2} = 5600k$

$\frac{\ln \frac{1}{2}}{5600} = k$

$A = A_0 e^{\left(\frac{\ln \frac{1}{2}}{5600}\right)t}$

$\frac{A}{A_0} = 15\%$

$.15 = e^{\left(\frac{\ln \frac{1}{2}}{5600}\right)t}$

$\ln .15 = \ln e^{\left(\frac{\ln \frac{1}{2}}{5600}\right)t}$

$\ln .15 = \frac{\ln \frac{1}{2}}{5600} \cdot t$

$\ln .15 \left(\frac{5600}{\ln \frac{1}{2}}\right) = t$

$15327 \text{ years} \approx$

HW: p447 # 2-34 even