4.1 (part 2) | Angles & Radian Measure

θ = theta
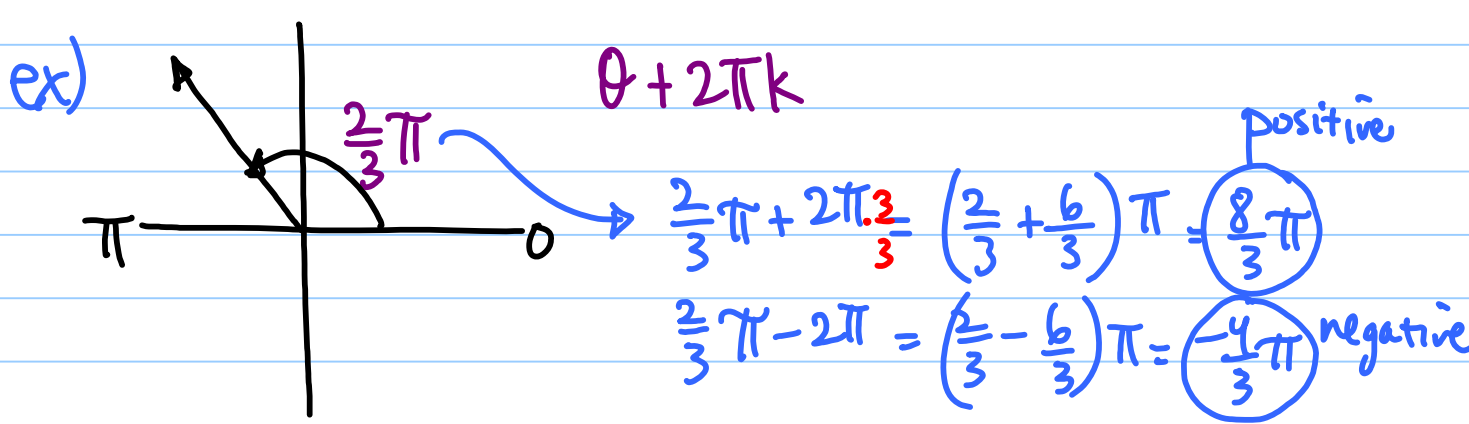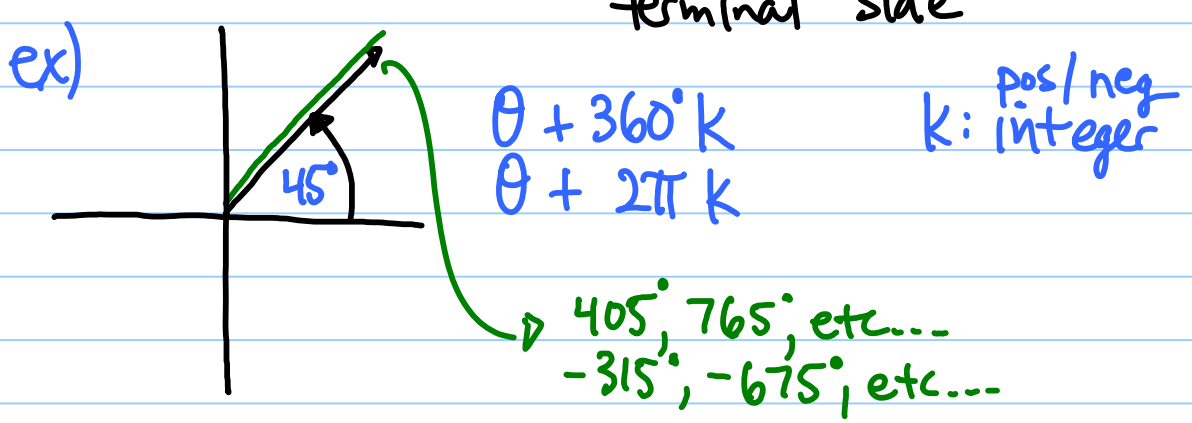(CCW: pos / CW: neg)

angle in standard position
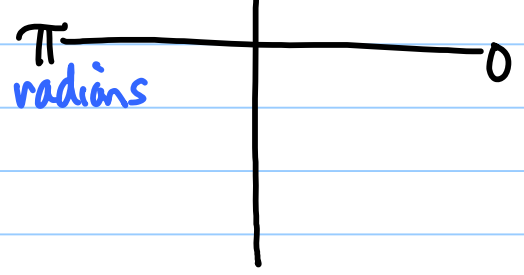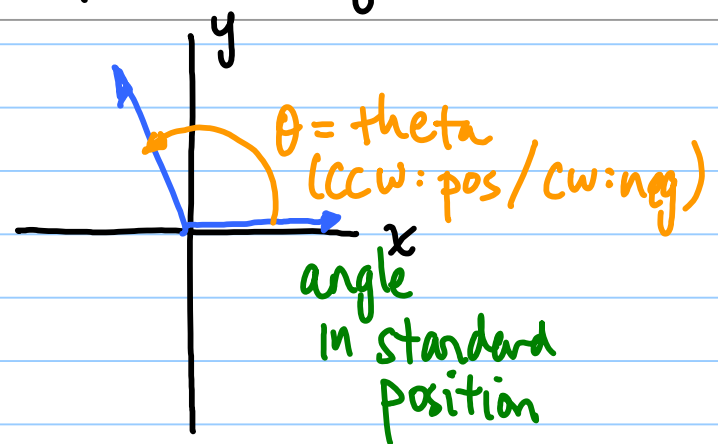
π radians

Degrees ⟷ Radians

$\longrightarrow \cdot \dfrac{\pi}{180}$

$\longleftarrow \cdot \dfrac{180}{\pi}$

Radian

$$\theta = \frac{\text{arc length}}{\text{radius}} = \frac{2\pi r}{r}$$

Coterminal Angles — angles that share the same terminal side

ex)

45°

$\theta + 360° k \qquad k: \text{pos/neg integer}$
$\theta + 2\pi k$

→ 405°, 765°, etc...
  −315°, −675°, etc...

ex)

$\frac{2}{3}\pi$

π $\qquad$ 0

$\theta + 2\pi k$

$\frac{2}{3}\pi + 2\pi \cdot \dfrac{3}{3} = \left(\dfrac{2}{3} + \dfrac{6}{3}\right)\pi = \boxed{\dfrac{8}{3}\pi}$ positive

$\frac{2}{3}\pi - 2\pi = \left(\dfrac{2}{3} - \dfrac{6}{3}\right)\pi = \boxed{-\dfrac{4}{3}\pi}$ negative

<u>Checkpoint</u>: Give one positive & one negative coterminal ∡.

1) $-115° \; +360° = \boxed{245°}$
   $\quad\quad\quad -360° = \boxed{-475°}$

2) $\dfrac{14\pi}{3} + 2\pi = \left(\dfrac{14}{3} + \dfrac{6}{3}\right)\pi = \boxed{\dfrac{20\pi}{3}}$
   $\quad\quad\;\; - 2\pi = \left(\dfrac{14}{3} - \dfrac{6}{3}\right)\pi = \boxed{\dfrac{8\pi}{3}}$

3) $-453° + 360° = \boxed{-93°}$ not enough
   $\begin{array}{r} 453 \\ -360 \\ \hline \end{array}$ $\begin{array}{r} +360 \\ \hline \boxed{267°} \end{array}$

4) $\dfrac{15\pi}{4} \rightarrow \left(3\tfrac{3}{4}\right)\pi$
   $\left(\cancel{3} + 1\tfrac{3}{4}\right)\pi$ -2π
   $= \boxed{-\dfrac{\pi}{4}}$

#

Draw Angles in Standard Position

ex) $\gamma = \dfrac{9\pi}{4}$

gamma

$\rightarrow \left(2\tfrac{1}{4}\right)\pi$
$\left(\cancel{2} + \tfrac{1}{4}\right)\pi$

π ———— 0
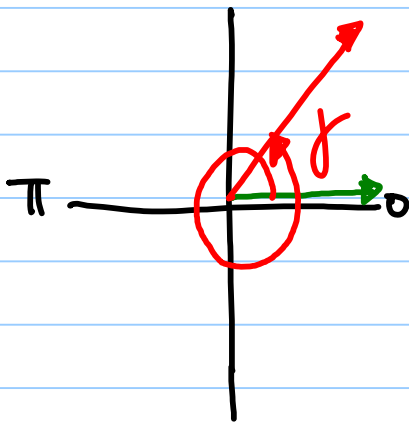
$\theta = \dfrac{s}{r}$ ← arc length
$\quad\quad\;$ ← radius

$s = r\theta$

↑ radian measure only

* if degrees then mult by $\dfrac{\pi}{180}$

ex) Find the radian measure of the central angle of a ⊙ of radius 14 cm and arc length 10 cm.
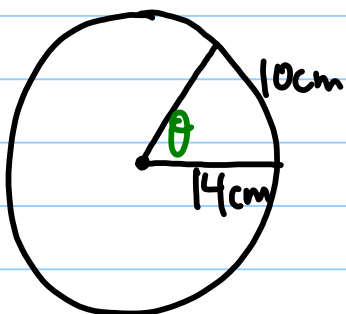
10cm
θ
14cm

$\theta = \dfrac{s}{r} = \dfrac{10\,cm}{14\,cm} = .71 \; radians$

↳ $40.9°$

ex) Find the arc length, given radius is 10 miles and central angle of 22°.

$s = r\theta$

$$S = (10 \text{ miles}) \, 22° \cdot \frac{\pi}{180°} = 3.84 \text{ miles}$$

p. 473, 2-12 even, 30-40 even, 41-74