10/23 TUE

5.1 (part 1) Verifying Trig Identities

Reciprocal

$\sin x = \frac{1}{\csc x}$ $\quad \csc x = \frac{1}{\sin x}$

$\cos x = \frac{1}{\sec x}$ $\quad \sec x = \frac{1}{\cos x}$

$\tan x = \frac{1}{\cot x}$ $\quad \cot x = \frac{1}{\tan x}$

Quotient

$\tan x = \frac{\sin x}{\cos x}$

$\cot x = \frac{\cos x}{\sin x}$

$x^2 + y^2 = r^2$

Pythagorean

$\sin^2 x + \cos^2 x = 1$

$\div \sin^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

$\div \cos^2 x$ $\quad \tan^2 x + 1 = \sec^2 x$

Even-Odd

even:
- $\cos(-x) = \cos x$
- $\sec(-x) = \sec x$

odd:
- $\sin(-x) = -\sin x$
- $\csc(-x) = -\csc x$
- $\tan(-x) = -\tan x$
- $\cot(-x) = -\cot x$

ex 1) Verify $\sec x \cot x = \csc x$

$\frac{1}{\cos x} \cdot \frac{\cos x}{\sin x}$

$\csc x$

$\frac{1}{\sin x}$

* Start w/ the more complicated side
* Rewrite into sines & cosines
* use identities

ex 2) Verify $\sin x \tan x + \cos x = \sec x$

$\sin x \cdot \frac{\sin x}{\cos x} + \cos x$

add the fractions

$\frac{\sin^2 x}{\cos x} + \frac{\cos x \cdot \cos x}{1 \cdot \cos x}$ * need a common denom...

$\frac{\sin^2 x + \cos^2 x}{\cos x}$ * pythag identity

$\frac{1}{\cos x}$

$\sec x$

$\frac{1}{\cos x}$

ex 3) Verify $\cos x - \cos x \sin^2 x = \cos^3 x$

* factor... $\cos x(1 - \sin^2 x)$

$\cos x \cdot \cos^2 x$

$\cos^3 x$

$\sin^2 x + \cos^2 x = 1$

$-\sin^2 x \qquad -\sin^2 x$

$\cos^2 x = 1 - \sin^2 x$

rearrange

ex 4) Verify $\frac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

Method #1

$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$\sec\theta + \tan\theta$

$\sec\theta = \frac{1}{\cos\theta}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Method #2

→ sines & cosines

ex 5) Verify $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 2\sec x$

$\sec x = \frac{1}{\cos x}$

* fractions → LCD $(1+\sin x)\cos x$

$\frac{\cos x}{(1+\sin x)} \cdot \frac{\cos x}{\cos x} + \frac{(1+\sin x)}{\cos x} \cdot \frac{(1+\sin x)}{(1+\sin x)}$

$\frac{\cos^2 x + 1 + \sin x + \sin x + \sin^2 x}{(1+\sin x)\cos x}$

$\frac{1 + 1 + 2\sin x}{(1+\sin x)\cos x}$

* Factor out the 2....

$\frac{2 + 2\sin x}{(1+\sin x)\cos x}$

$\frac{2(1+\sin x)}{(1+\sin x)\cos x}$

$\frac{2}{\cos x}$

$2 \cdot \sec x$

ex 6) Verify $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$.

binomial

* mult by the conjugate → $ax+b$ & $ax-b$; $\sqrt{\ }$ & i

FOIL → $\frac{\sin x}{(1+\cos x)} \cdot \frac{(1-\cos x)}{(1-\cos x)}$

$\frac{\sin x(1-\cos x)}{1-\cos x+\cos x-\cos^2 x}$

$\frac{\sin x(1-\cos x)}{1-\cos^2 x}$

Pythag Id → $1-\cos^2 x$

$\frac{\sin x(1-\cos x)}{\sin^2 x}$

$\frac{1-\cos x}{\sin x}$

ex 7) $\frac{\tan x - \sin(-x)}{1+\cos x} = \tan x$

odd

$\tan x = \frac{\sin x}{\cos x}$

$\frac{\tan x - -\sin x}{1+\cos x}$

$\frac{\tan x + \sin x}{1+\cos x}$ → sines & cosines

$\frac{\frac{\sin x}{\cos x} + \frac{\sin x}{1}}{1+\cos x}$ → combine: LCD $\cos x$

$\frac{\frac{\sin x}{\cos x} + \frac{\sin x}{1} \cdot \frac{\cos x}{\cos x}}{1+\cos x}$

$\frac{\left(\frac{\sin x + \sin x \cos x}{\cos x}\right)}{\left(\frac{1+\cos x}{1}\right)}$

mult by the reciprocal

$$\frac{(\sin x + \sin x \cos x)}{\cos x} \cdot \frac{1}{(1+\cos x)}$$

factor out $\sin x$

$$\frac{\sin x(1+\cos x)}{\cos x} \cdot \frac{1}{(1+\cos x)}$$

ex 8) Verify $\frac{\tan x}{\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta}} = 2 + 2\cot^2\theta$

Find LCD: $(1+\cos\theta)(1-\cos\theta)$

$$\frac{1}{(1+\cos\theta)}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)} + \frac{1}{(1-\cos\theta)}\cdot\frac{(1+\cos\theta)}{(1+\cos\theta)}$$

FOIL → $\frac{1-\cos\theta+1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)}$

$$\frac{2}{1-\cos\theta+\cos\theta-\cos^2\theta}$$

Pythag ID → $\frac{2}{1-\cos^2\theta}$

$$\frac{2}{\sin^2\theta}$$

"Uh oh!"

$2(1+\cot^2\theta)$

$2\cdot\csc^2\theta$

$\frac{2}{\sin^2\theta}$

Bridge together