10/29 MON

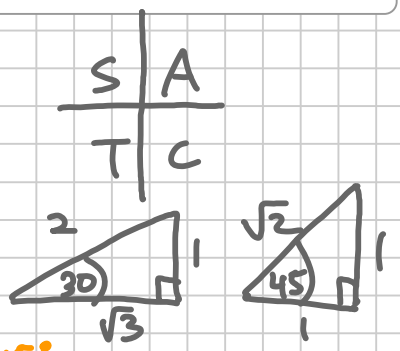
5.2 Sum & Diff Formulas

0, 30°, 45°, 60°, 90°, etc...

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$



ex 1) Find the exact value of $\cos 15°$.

$15° = 60° - 45°$

$\cos 15° = \cos(60° - 45°)$

$= \cos 60°\cos 45° + \sin 60°\sin 45°$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

0.9659...

ex 2) Find the exact value of $\cos 80°\cos 20° + \sin 80°\sin 20°$

$\cos(\alpha - \beta)$

$\alpha = 80°$, $\beta = 20°$

$\cos(80° - 20°) = \cos 60° = \frac{1}{2}$

ex 3) Verify $\frac{\cos(\alpha-\beta)}{\sin\alpha\cos\beta} = \cot\alpha + \tan\beta$

$$\frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\cos\beta}$$

$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

$$\frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta}$$

$$\cot\alpha + \tan\beta$$

ex 4) Find $\sin\frac{7\pi}{12}$ (Note: $\frac{7\pi}{12} =$)

$\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$

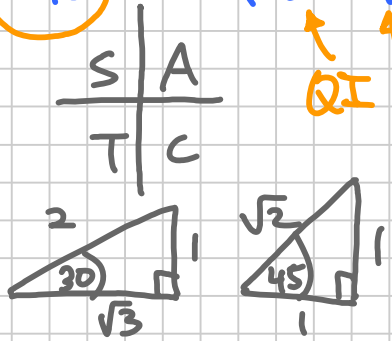
$105° \leftarrow 45° + 60°$

$\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

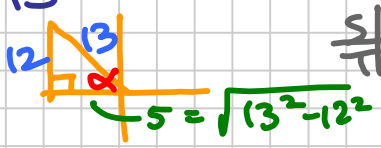
$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

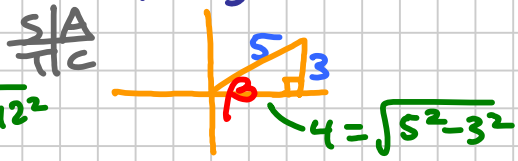
QII, QI, QI



ex 5) Suppose $\sin\alpha = \frac{12}{13}$ in QII & $\sin\beta = \frac{3}{5}$ in QI

a) $\cos\alpha = \frac{-5}{13}$





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b) $\cos\beta = \frac{4}{5}$

c) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\frac{-5}{13}\cdot\frac{4}{5} - \frac{12}{13}\cdot\frac{3}{5} = \frac{-20-36}{65} = \frac{-56}{65}$

d) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

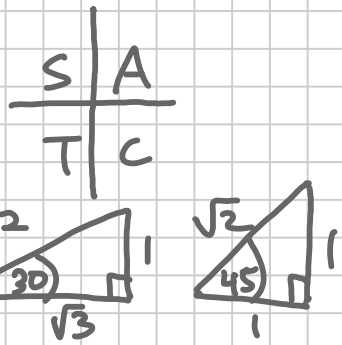
$\frac{12}{13}\cdot\frac{4}{5} + \frac{-5}{13}\cdot\frac{3}{5} = \frac{48-15}{65} = \frac{33}{65}$

ex 7) Verify $\tan\left(x-\frac{\pi}{4}\right) = \frac{\tan x - 1}{\tan x + 1}$

$\frac{\tan x - \tan\frac{\pi}{4}}{1+\tan x\cdot\tan\frac{\pi}{4}}$

45°

$\frac{\tan x - 1}{1+\tan x\cdot 1}$



p 603 # 2-64 EOE (skip 54)

... 50, ~~54~~, 56, 60, 64

..... $\frac{3-4\sqrt{5}}{2+\sqrt{7}}$ rationalize $\cdot\frac{2-\sqrt{7}}{2-\sqrt{7}}$