11/1 THU

5.3 Double-Angle, Power-Reducing, & Half-Angle Formulas

Double Angle

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$ ← replaced $\sin^2\theta$

$= 1 - 2\sin^2\theta$ ← replaced $\cos^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Power Reducing

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$

Half-Angle

$\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

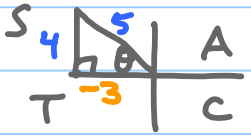
$\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$

$*\tan\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

$= *\dfrac{1-\cos\alpha}{\sin\alpha} = *\dfrac{\sin\alpha}{1+\cos\alpha}$

ex 1) $\sin\theta = \frac{4}{5}$, θ is in QII

Find all the double angles



a) $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cdot\frac{4}{5}\cdot\frac{-3}{5} = \frac{-24}{25}$

b) $\cos 2\theta = 2\cos^2\theta - 1$

$= 2\cdot\left(\frac{-3}{5}\right)^2 - 1 = 2\cdot\frac{9}{25} - 1 = \frac{18}{25} - 1 = \frac{18-25}{25} = \frac{-7}{25}$

c) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot\left(\frac{4}{-3}\right)}{1-\left(\frac{4}{-3}\right)^2} = \frac{\frac{8}{-3}}{1-\frac{16}{9}} = \frac{\frac{8}{-3}}{\frac{9-16}{9}} = \frac{8}{-3}\cdot\frac{9}{-7} = \frac{72}{21} = \frac{24}{7}$

ex 2) $\frac{2\tan 15^\circ}{1-\tan^2 15^\circ} = \tan(2\cdot 15^\circ) = \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

ex 3) Verify $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

sum

$\cos(2\theta+\theta)$

double ∠s

$\cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$(2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$

Pythag

$2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$

$2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$

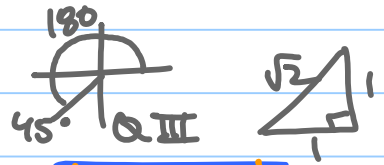
$2\cos^3\theta - \cos\theta - 2 + 2\cos^2\theta\cos\theta$

$2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$4\cos^3\theta - 3\cos\theta$

ex 5) $\cos 112.5^\circ$ — double: 225°

$\frac{1}{2}$ angle $\cos\frac{225}{2} = \pm\sqrt{\frac{1+\cos 225}{2}} = \pm\sqrt{\frac{1+\left(\frac{-\sqrt{2}}{2}\right)}{2}}$



$= \pm\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{\frac{2}{1}}} = \pm\sqrt{\frac{2-\sqrt{2}}{2}\cdot\left(\frac{1}{2}\right)} = \pm\sqrt{\frac{2-\sqrt{2}}{4}}$

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