

$$\frac{\text{S}}{\text{T}} = \frac{\text{A}}{\text{C}}$$

6.3 (part 2) | Polar Coordinates

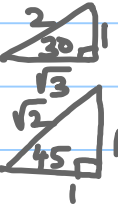
Rectangular coordinates (x, y) \longleftrightarrow Polar coordinates (r, θ)

$$x^2 + y^2 = r^2$$

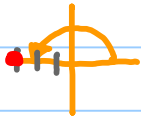
$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\tan \theta = \frac{y}{x}$$

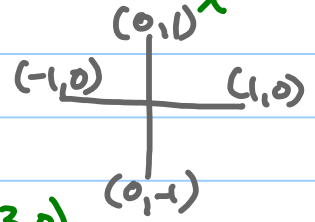


ex 1) Convert to rect. coords: $(3, \pi)$



$$x = r \cos \theta = 3 \cos \pi = 3 \cdot (-1) = -3$$

$$y = r \sin \theta = 3 \sin \pi = 3(0) = 0$$



ex 2) Convert to rect. coords: $(-10, \frac{\pi}{6})$

$$x = r \cos \theta = -10 \cos \frac{\pi}{6} = -10 \left(\frac{\sqrt{3}}{2}\right) = -5\sqrt{3}$$

$$y = r \sin \theta = -10 \sin \frac{\pi}{6} = -10 \left(\frac{1}{2}\right) = -5$$

Q III

$$\therefore (-5\sqrt{3}, -5)$$

ex 3) Convert to polar coords: $(1, -\sqrt{3})$

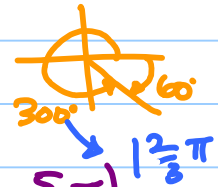
$$x^2 + y^2 = r^2 \Rightarrow (1)^2 + (-\sqrt{3})^2 = r^2 \Rightarrow 1 + 3 = r^2 \Rightarrow 2 = r$$

$$\tan \theta = \frac{y}{x} = \frac{-\sqrt{3}}{1} \Rightarrow \theta = \tan^{-1}(-\sqrt{3})$$

* $\theta_{\text{ref}} = 60^\circ$

Q IV

$$\therefore (2, \frac{5}{3}\pi)$$



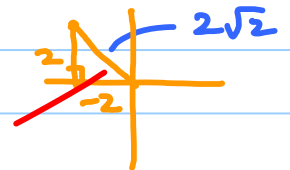
* ex 4) Convert to polar coords: $(-2, 2)$

\rightarrow special Δ .

or ... $x^2 + y^2 = r^2 \Rightarrow (-2)^2 + (2)^2 = r^2 \Rightarrow 4 + 4 = r^2 \Rightarrow 2\sqrt{2} = r$

$$\tan \theta = \frac{y}{x} = \frac{2}{-2} = -1 \Rightarrow \theta = \frac{3}{4}\pi$$

$$\therefore (2\sqrt{2}, \frac{3}{4}\pi)$$



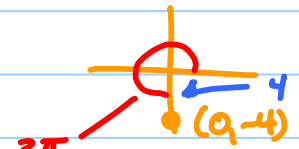
* ex 5) Convert to polar coords: $(0, -4)$

\rightarrow quadrantal Δ

or ... $x^2 + y^2 = r^2 \Rightarrow (0)^2 + (-4)^2 = r^2 \Rightarrow 16 = r^2 \Rightarrow 4 = r$

$$\tan \theta = \frac{y}{x} = \frac{-4}{0} \Rightarrow \theta = \frac{3}{2}\pi$$

$$\therefore (4, \frac{3}{2}\pi)$$



Equations : Rect \leftrightarrow Polar



$$x^2 + y^2 = r^2$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\tan \theta = \frac{y}{x}$$

ex 6) Convert to polar : $5x - y = 6$ (line)

$$5(r \cos \theta) - (r \sin \theta) = 6$$

$$5r \cos \theta - r \sin \theta = 6$$

$$r(5 \cos \theta - \sin \theta) = 6$$

$$r = \frac{6}{5 \cos \theta - \sin \theta}$$

ex 7) Convert to Polar :

Circle $\rightarrow x^2 + (y+1)^2 = 1$

$$(r \cos \theta)^2 + (r \sin \theta + 1)^2 = 1$$

$$r^2 \cos^2 \theta + (r \sin \theta + 1)(r \sin \theta + 1) = 1$$

$$* r^2 \cos^2 \theta + r^2 \sin^2 \theta + 2r \sin \theta + 1 = 1$$

$$r^2(\cos^2 \theta + \sin^2 \theta) + 2r \sin \theta = 0$$

$$r^2 + 2r \sin \theta = 0$$

$$r(r + 2 \sin \theta) = 0$$

$r = 0$ or $r + 2 \sin \theta = 0$

$$r = -2 \sin \theta$$

factor ... product of factors

ex 8) Convert to Rect.:

$$r = 4$$

subst $\rightarrow r^2 = 4^2$ Square both sides

$$x^2 + y^2 = 16$$

ex 9) Convert to Rect.:

$$\theta = \frac{3\pi}{4}$$

tan both sides...

$$\tan \theta = \tan \frac{3\pi}{4}$$

subst

$$\tan \theta = -1$$

$$x \cdot \frac{y}{x} = -1 \cdot x$$

$$y = -x$$

ex 10) Convert to Rectangular

$$r = -2 \sec \theta$$

$\cos \theta \cdot r = \frac{-2}{\cos \theta} \cdot \cos \theta$ Mult both sides

Subst $\rightarrow r \cos \theta = -2$

$$x = -2$$

HW = p 673, #34-74 even