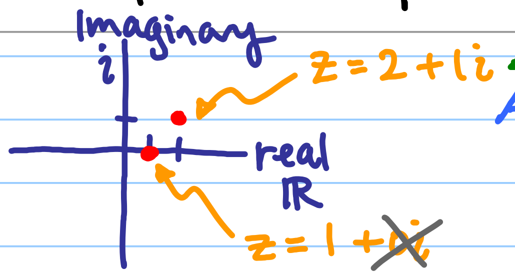
12/19 WED

real imaginary
$z = a + bi$

6.5 (part 1) Complex Numbers in Polar Form; DeMoivre's Theorem

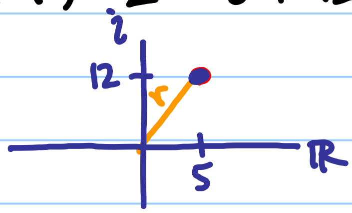


"(2,1)"

Absolute Value (find the distance to origin)

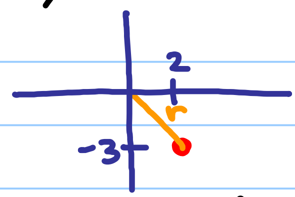
$|z| = |a+bi| = \sqrt{a^2+b^2}$ → Pythag. Thm

ex 1) $z = 5 + 12i$. Plot & find the absolute value.



$|z| = |5+12i| = \sqrt{5^2+12^2} = 13$ units

ex 2) $z = 2 - 3i$. Plot & find $|z|$.

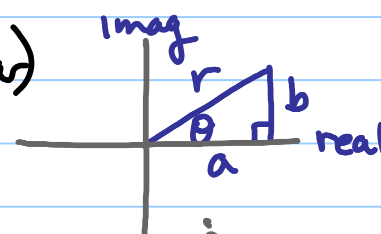


$|z| = |2-3i| = \sqrt{2^2+(-3)^2} = \sqrt{13}$

$z = a + bi$ (rect) ⟷ $r(\cos\theta + i\sin\theta)$ (polar)

$a = r\cos\theta$ $\quad r = \sqrt{a^2+b^2}$

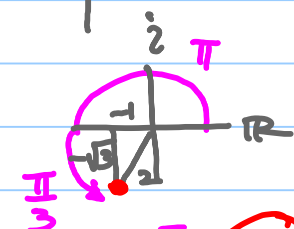
$b = r\sin\theta$ $\quad \tan\theta = \frac{b}{a}$



ex 3) Convert to Polar: $z = -1 - i\sqrt{3}$ (r,θ ; $a + bi$)

$r = \sqrt{a^2+b^2} = \sqrt{(-1)^2+(-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\theta_{ref} = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 60°$ or $\frac{\pi}{3}$



$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$ or $240°$

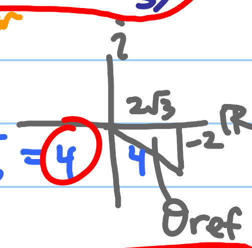
QIII

→ $r(\cos\theta + i\sin\theta) = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$ polar

ex4) Convert to Polar: $z = 2\sqrt{3} - 2i$ (a, b)

$r = \sqrt{a^2+b^2} = \sqrt{(2\sqrt{3})^2+(-2)^2} = \sqrt{12+4} = \sqrt{16} = 4$

$\theta_{ref} = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{2}{2\sqrt{3}}\right) = 30°$



QIV

360 − 30

$\theta: 2\pi - \frac{\pi}{6}$

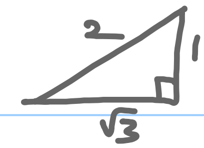
$\theta: \frac{11}{6}\pi$

→ $r(\cos\theta + i\sin\theta) = 4\left(\cos\frac{11}{6}\pi + i\sin\frac{11}{6}\pi\right)$

or $4\,\text{cis}\,\frac{11\pi}{6}$

Polar $\rightarrow$ rect $z = a + bi$

$a = r\cos\theta$
$b = r\sin\theta$



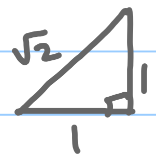
ex 5) $z = \overset{r}{4}\left(\cos \overset{\theta}{30^\circ} + i \sin \overset{\theta}{30^\circ}\right)$ θ: QI

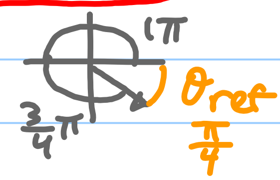
$a = r\cos\theta = 4\cos 30^\circ = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$b = r\sin\theta = 4\sin 30^\circ = 4\left(\frac{1}{2}\right) = 2$ $\rightarrow$ $z = 2\sqrt{3} + 2i$

ex 6) $z = 2\sqrt{3}\left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}\right)$ θ : QIV

S | A
T | C





$a = r\cos\theta = 2\sqrt{3}\cos 45^\circ = \cancel{2}\sqrt{3}\left(\frac{\sqrt{2}}{\cancel{2}}\right) = \sqrt{6}$

$b = r\sin\theta = 2\sqrt{3}\sin 45^\circ = \cancel{2}\sqrt{3}\left(\frac{\sqrt{2}}{\cancel{2}}\right) = -\sqrt{6}$ QIV: sin negative

$\rightarrow$ $z = \sqrt{6} - \sqrt{6}i$

Product of Complex #s in Polar Form

$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ & $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$$z_1 \cdot z_2 = r_1 r_2\left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

Quotient of Complex #s in Polar Form

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

ex 7) $z_1 = 6\left[\cos 40^\circ + i\sin 40^\circ\right]$
$z_2 = 5\left[\cos 20^\circ + i\sin 20^\circ\right]$

a) Find $z_1 \cdot z_2 = 6 \cdot 5\left[\cos(40^\circ + 20^\circ) + i\sin(40^\circ + 20^\circ)\right]$
$= 30\left[\cos 60^\circ + i\sin 60^\circ\right]$

b) Find $\frac{z_1}{z_2} = \frac{6}{5}\left[\cos(40^\circ - 20^\circ) + i\sin(40^\circ - 20^\circ)\right]$
$= \frac{6}{5}\left[\cos 20^\circ + i\sin 20\right]$

Hw, p696 #2-52 even