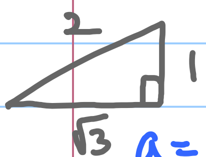
12/21 FRI ← LOL

6.5 (part 2) .....

DeMoivre's Theorem (powers)

$$z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

ex 1) $[2\cos 20^\circ + i\sin 20^\circ]^3$



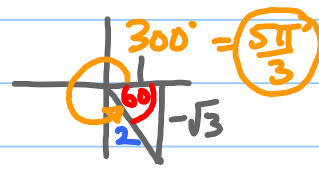
$= 2^3(\cos(3\cdot 20) + i\sin(3\cdot 20))$

$= 8(\cos 60^\circ + i\sin 60)$ ← polar complex

$a = r\cos\theta = 8\cos 60 = 8\cdot\frac{1}{2} = 4$ ← continue to rectangular

$b = r\sin\theta = 8\sin 60^\circ = 8\cdot\frac{\sqrt{3}}{2} = 4\sqrt{3}$ → $4 + 4\sqrt{3}i$

ex 2) $(1 - i\sqrt{3})^7$



$r = \sqrt{a^2 + b^2}$

$= \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2$

S | A
T | C

$\left[2\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)\right]^7 = 2^7\left(\cos 7\cdot\frac{5\pi}{3} + i\sin 7\cdot\frac{5\pi}{3}\right)$

$= 128\ \text{cis}\ \frac{35\pi}{3} = 3\overline{)35}\ 11 = 11\frac{2}{3} = 1\frac{2}{3}\pi = \frac{5\pi}{3}$ R 2

polar → rect.

$\Rightarrow a = 128\cos\frac{5\pi}{3} = 128\cdot\frac{1}{2} = 64$

$b = 128\sin\frac{5\pi}{3} = 128\left(-\frac{\sqrt{3}}{2}\right) = -64\sqrt{3}$ → $64 - 64\sqrt{3}i$

sin 60 QIV

Roots of Complex #s in Polar Form

$$z_k = \sqrt[n]{r}\left[\cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}\right]$$

or 360°k

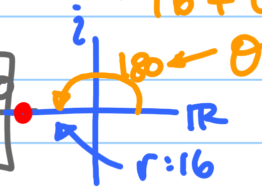
$k = 0, 1, 2, 3, \ldots$

ex 3) Find all the complex square roots of $\sqrt[2]{\ }$ $49(\cos 50^\circ + i\sin 50^\circ)$. Leave answer in polar.

$k=0: \longrightarrow \sqrt[2]{49}\left[\cos\frac{50+\cancel{360\cdot 0}}{2} + i\sin\frac{50+\cancel{360\cdot 0}}{2}\right] = 7[\cos 25^\circ + i\sin 25]$

$k=1: \longrightarrow \sqrt{49}\left[\cos\frac{50+360\cdot 1}{2} + i\sin\frac{50+360\cdot 1}{2}\right] = 7[\cos 205^\circ + i\sin 205^\circ]$

ex 4) Complex 4th roots of -16. rect: $-16+0i$



$k=0: \longrightarrow \sqrt[4]{16}\left[\cos\frac{180+360\cdot 0}{4} + i\sin\frac{180+360\cdot 0}{4}\right]$ r

$= 2(\cos 45^\circ + i\sin 45^\circ) = 2\cdot\frac{\sqrt{2}}{2} + 2\cdot\frac{\sqrt{2}}{2}i = \sqrt{2}+\sqrt{2}i$

$k=1: \longrightarrow \sqrt[4]{16}\left[\cos\frac{180+360\cdot 1}{4} + i\sin\frac{180+360\cdot 1}{4}\right] = 2\,\text{cis}\,\frac{540}{4} = 2\,\text{cis}\,135$

$= -\sqrt{2}+\sqrt{2}i$ QII S|A T|C

$k=2: \longrightarrow \sqrt[4]{16}\left[\cos\frac{180+360\cdot 2}{4} + i\sin\frac{180+360\cdot 2}{4}\right] = 2\,\text{cis}\,\frac{900}{4} = 2\,\text{cis}\,225$

$= -\sqrt{2}-\sqrt{2}i$ QIII S|A T|C

$k=3: \longrightarrow \sqrt[4]{16}\left[\cos\frac{180+360\cdot 3}{4} + i\sin\frac{180+360\cdot 3}{4}\right] = 2\,\text{cis}\,\frac{1260}{4} = 2\,\text{cis}\,315$

$= \sqrt{2}-\sqrt{2}i$ QIV S|A T|C

HW: p696, #54, 56, 58, 62, 66, 72