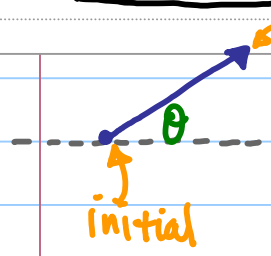
1/7 MON

6.6 & 6.7 Vectors & The Dot Product



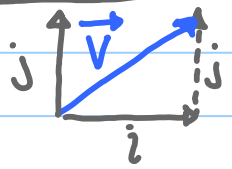
magnitude = length $\|v\| = \sqrt{a^2+b^2}$

direction = angle $m = \frac{y_2-y_1}{x_2-x_1}$

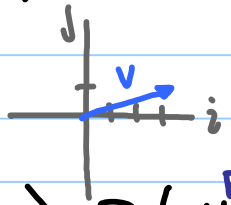
"equal vectors": same magnitude & direction

opposite vectors: " " & opposite direction

unit vector: $v = ai + bj$ ← not imaginary #



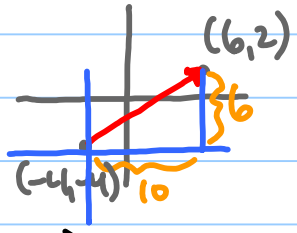
ex 1) Draw the vector & Find the magnitude: $\vec{v} = 3\vec{i} + \vec{j}$



$\|v\| = \sqrt{a^2+b^2} = \sqrt{3^2+1^2} = \sqrt{10}$

ex 2) $P_1(-4,-4)$, $P_2(6,2)$ ← coordinates — Find both vectors.

"ray" $\overrightarrow{P_1P_2} \neq \overrightarrow{P_2P_1}$ → opposite vectors



$\overrightarrow{P_1P_2}$: $v = (x_2-x_1)i + (y_2-y_1)j$

$= (6--4)i + (2--4)j$

$= 10i + 6j$

$\overrightarrow{P_2P_1}$: $v = (-4-6)i + (-4-2)j$

$= -10i - 6j$

ex 3) $v_1 = (1,2),(4,6)$ & $v_2 = (0,0)(5,4)$ Are the vectors equal?

$v_1 = (4--1)i + (6-2)j$

$= 5i + 4j$ → yes.... or Compare magnitude & slope

$v_2 = (5-0)i + (4-0)j = 5i + 4j$

ex 4) $v = 3i+2j$, $w = 5i+2j$, $u = 6i-4j$

a) $v+w$?

$3i+2j+5i+2j$

→ $8i+4j$

b) $6u$ ← scalar multiplier

$6(6i-4j)$

→ $36i-24j$

c) $2v-3w$

$2(3i+2j)-3(5i+2j) = 6i+4j-15j-6j$

→ $-9i-2j$

Unit Vector: $\frac{v}{\|v\|}$ ← vector; ← magnitude

ex) 100 miles in 4 hours... $\frac{100 \text{ miles}}{4 \text{ hrs}} = 25$ mph

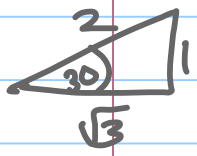
ex 5) $v = 2i - 3j$. Find the unit vector.

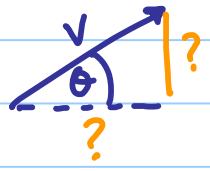
$\|v\| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$ → unit vector: $\frac{2i-3j}{\sqrt{13}} = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} i - \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} j$ (rationalize)

x & y components

$= \frac{2\sqrt{13}}{13} i - \frac{3\sqrt{13}}{13} j$

ex 6) Write the unit vector given the $\|v\|$ and direction. → $v = \|v\|\cos\theta\, i + \|v\|\sin\theta\, j$





$\|v\| = 6$ & $\theta = 30°$

$v = 6\cos 30° i + 6\sin 30° j = 6\left(\frac{\sqrt{3}}{2}\right) i + 6\left(\frac{1}{2}\right) j$

$= 3\sqrt{3} i + 3j$

Dot Product → multiply two vectors → scalene

$v: a_1 i + b_1 j$

$w: a_2 i + b_2 j$

$v \cdot w = a_1 \cdot a_2 + b_1 \cdot b_2$ (dot; number)

ex 7) $v = 7i - 4j$; $w = 2i - j$; $u = 2i + 3j$

a) Find $v \cdot w$.

$v \cdot w = 7(2) + (-4)(-1) = 14 + 4 = 18$

Find the angle between two vectors.

$\cos\theta = \frac{v \cdot w}{\|v\|\,\|w\|}$

ex 8) $v = 3i - 2j$ & $w = -i + 4j$. Find the angle between the vectors.

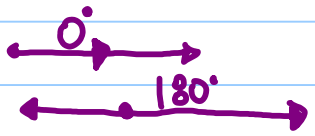
$\cos\theta = \frac{3(-1) + (-2)(4)}{\sqrt{3^2 + (-2)^2} \cdot \sqrt{(-1)^2 + (4)^2}} = \frac{-11}{\sqrt{221}}$

$\theta = \cos^{-1}\left(\frac{-11}{\sqrt{221}}\right) = 137.73°$

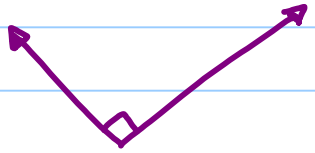
"Orthogonal"

Parallel & Perpendicular Vectors

The angle between the vectors = 0° or 180°



The dot product is 0.



ex 9) $v = 3i + 4j$ & $w = 6i + 8j$. Are they parallel?

$\|v\| = \sqrt{3^2+4^2}$ $\quad \|w\| = \sqrt{6^2+8^2}$

$\|v\| = 5$ $\quad = 10$

$$\cos\theta = \frac{v \cdot w}{\|v\| \cdot \|w\|} = \frac{3(6)+4(8)}{5 \cdot 10} = \frac{50}{50}$$

$\cos\theta = 1$

$\theta = \cos^{-1}(1)$

$\theta = 0$

$\therefore \vec{v} \parallel \vec{w}$

ex 10) $v = 6i - 3j$ & $w = i + 2j$. Are they orthogonal? ($\perp$)

→ Check if $v \cdot w = 0$.

$v \cdot w = (6)(1) + (-3)(2) = 6 - 6 = 0$

$\therefore \vec{v} \perp \vec{w}$

HW

p 709, #6-50 (EOE)

p 719, #2-32 (EOE)