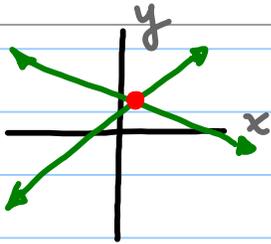
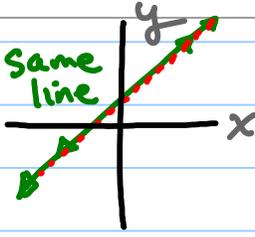


7.1 & 7.2 Solving Systems of Equations (Two/Three Variables) (Linear)



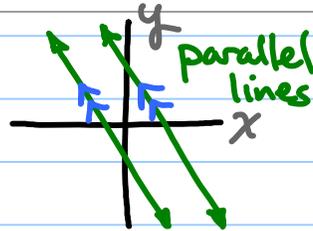
one solution

- consistent
- independent



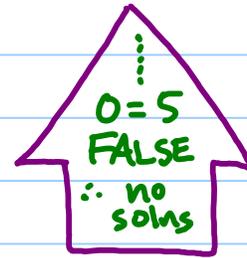
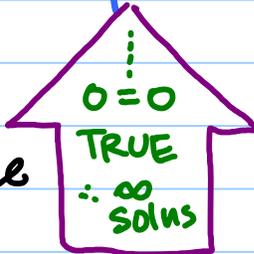
∞ solutions

- consistent
- dependent



no solutions, \emptyset

- inconsistent



Substitution

- isolate a variable & substitute into the other equation....

ex 1) $5x - 4y = 9$
Substitution → $x = 2y - 3$
already isolated

$$\begin{aligned} \rightarrow 5(2y - 3) - 4y &= 9 \\ 10y - 15 - 4y &= 9 \\ 6y &= 24 \end{aligned}$$

$$\begin{aligned} x &= 2(4) - 3 \\ x &= 5 \end{aligned}$$

$$y = 4$$

∴ (5, 4) : consistent & independent

Elimination

- line up $Ax + By = C$
- Add / Subt multiples of one equation to another
- "Add opposites / Subt same" : positive / negative

ex 2) $3x + 2y = 48$
Elimination $9x - 8y = -24$
opposites

need LCM
 $2 \text{ \& } -8 \rightarrow "8"$

$$\begin{aligned} (3x + 2y = 48) &\rightarrow 12x + 8y = 192 \\ 9x - 8y = -24 &\rightarrow (+) 9x - 8y = -24 \\ \hline 21x &= 168 \\ \underline{21} & \end{aligned}$$

$$\begin{aligned} x &= 8 \\ 3(8) + 2y &= 48 \\ 24 + 2y &= 48 \\ 2y &= 24 \\ y &= 12 \end{aligned}$$

∴ (8, 12) : consistent & independent

ex 3) $6x + 2y = 7$
 Elim or Subst? $y = 2 - 3x$

$6x + 2(2 - 3x) = 7$
 $6x + 4 - 6x = 7$
 $4 = 7$
 false

∴ no solution, inconsistent

ex 4) $(3x - y = 5) \cdot 7$ LCM
 Elim $21x - 7y = 35$ 21

$21x - 7y = 35$
 $(-)\ 21x - 7y = 35$
 $0 = 0$
 True

∴ ∞ solutions, same line, consistent, dependent

ex 5) $\begin{cases} ①\ 2x + y - 2z = -1 \\ ②\ 3x - 3y - z = 5 \\ ③\ x - 2y + 3z = 6 \end{cases}$

eliminate a variable: z



$① \& ②\ 2x + y - 2z = -1$ LCM 2
 $(3x - 3y - z = 5) \cdot 2$

$9x - 9y - 3z = 15$

$(+)\ x - 2y + 3z = 6$
 $(-)\ 9x - 9y - 3z = 15$
 $(+)\ 10x - 11y = 21$

$2x + y - 2z = -1$
 $(+)\ -6x + 6y + 2z = -10$
 $(+)\ -4x + 7y = -11$

$(-)\ -4x + 7y = -11$ LCM 20
 $(+)\ 10x - 11y = 21$

$-20x + 35y = -55$

$(+)\ 20x - 22y = 42$

$13y = -13$
 $y = -1$

$-4x + 7(-1) = -11$
 $-4x - 7 = -11$
 $-4x = -4$
 $x = 1$

Use #3 to solve z:

$(1) - 2(-1) + 3z = 6$

$1 + 2 + 3z = 6$

$3z = 3 \dots z = 1$

∴ $(1, -1, 1)$