4/1 MON

7.3 Partial Fractions

$\frac{1\cdot4}{3\cdot4}+\frac{1\cdot3}{4\cdot3}=\frac{7}{12}$ LCD 12

ex) $\frac{1}{(x+1)}+\frac{3}{(x-2)}$ Find the LCD: $(x+1)\cdot(x-2)$

adding rational expressions

$\frac{1}{(x+1)}\cdot\frac{(x-2)}{(x-2)}+\frac{3}{(x-2)}\cdot\frac{(x+1)}{(x+1)}=\frac{x-2+3x+3}{LCD}$

$=\frac{4x+1}{(x+1)(x-2)}$ OR x^2-x-2

Scenarios

ex) $\frac{9x^2-9x+6}{(2x-1)(x+2)(x-2)}=\frac{A}{2x-1}+\frac{B}{x+2}+\frac{C}{x-2}$

* Partial Fractions: # of factors (denominator)

ex) $\frac{x+2}{x^2+x-6}=\frac{A}{x+3}+\frac{B}{x-2}$ $x^2+x-6=(x+3)(x-2)$

ex) $\frac{x-3}{x^2}=\frac{A}{x}+\frac{B}{x^2}$ repeated factor $x\cdot x$ *

ex) $\frac{x^2-6x+3}{(x+3)^3}=\frac{A}{x+3}+\frac{B}{(x+3)^2}+\frac{C}{(x+3)^3}$ repeated factor

ex) $\frac{x^2+2x-3}{(x+1)(x^2-25)}=\frac{A}{x+1}+\frac{B}{(x+5)}+\frac{C}{(x-5)}$ $x^2-25=(x+5)(x-5)$

ex) $\frac{5x-1}{(x-3)(x+4)}=\frac{A}{(x-3)}+\frac{B}{(x+4)}$

* Multiply by LCD (mental math)

$\frac{5x-1}{(x-3)(x+4)}\cdot(x-3)(x+4)=\frac{A}{(x-3)}\cdot(x-3)(x+4)+\frac{B}{(x+4)}\cdot(x-3)(x+4)$

$5x-1=Ax+4A+Bx-3B$

Const: ~~ "x" ~~

$-1=4A-3B$

$(5=A+B)\cdot3$

$15=3A+3B$

$14=7A$

$A=2$

$B=3$

$\therefore \frac{2}{x-3}+\frac{3}{x+4}$

* System of Eqns: Set the c, x, x^2, ...

ex 2) $\frac{4x^2+13x-9}{x^3+2x^2-3x} = \frac{A}{x} + \frac{B}{(x+3)} + \frac{C}{(x-1)}$ * Mult by LCD

$x(x^2+2x-3)$

$x(x+3)(x-1)$ ← LCD

$4x^2+13x-9 = A(x+3)(x-1) + Bx(x-1) + Cx(x+3)$

$4x^2+13x-9 = Ax^2+2Ax-3A+Bx^2-Bx+Cx^2+3Cx$

System:

x^2: $4 = A+B+C \rightarrow 4 = 3+B+C \rightarrow 1 = B+C$ $\quad B=-1$

x: $13 = 2A-B+3C \rightarrow 13 = 2(3)-B+3C$

$+\ 7 = -B+3C$

$8 = 4C$

$2 = C$

C: $-9 = -3A$

$A = 3$

$\frac{3}{x} + \frac{-1}{x+3} + \frac{2}{x-1}$

ex 3) $\frac{6x-11}{x^2-2x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$ * Mult by LCD

$(x-1)(x-1)$

repeated factor → $(x-1)^2$ ← LCD

$6x-11 = A(x-1) + B$

$6x-11 = Ax-A+B$

x: $6 = A$

C: $-11 = -A+B \quad B=-5$

$\therefore \frac{6}{x-1} + \frac{-5}{(x-1)^2}$