4/2 TUE

7.3 (part 2) Partial Fractions → Prime Quadratic (x^2) Factors

- Linear, constant factors
  ex) $2x+1$, 6, etc...

ex 1) $\dfrac{5x^2-6x+7}{(x-1)(x^2+1)} = \dfrac{A}{(x-1)} + \dfrac{Bx+C}{x^2+1}$

LCD: $(x-1)(x^2+1)$ — prime quad factor: x^2+1

* Mult by LCD

FOIL

$5x^2-6x+7 = A(x^2+1)+(Bx+C)(x-1)$

$5x^2-6x+7 = Ax^2+A+Bx^2-Bx+Cx-C$

$B = 5-A$ (Substitution)

$x^2: 5 = A+B$

$x: -6 = -B+C \rightarrow -6 = -(5-A)+C \rightarrow -6 = -5+A+C$

$\#: 7 = A-C$

$-1 = A+C$

$(+)\ 7 = A-C$

$6 = 2A$

mental math ←

$A = 3$

$B = 2$

$C = -4$

$\therefore \dfrac{3}{x-1} + \dfrac{2x-4}{x^2+1}$

ex 2) $\dfrac{5x^2+6x+3}{(x+1)(x^2+2x+2)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+2x+2}$

FOIL

$5x^2+6x+3 = A(x^2+2x+2)+(Bx+C)(x+1)$

$5x^2+6x+3 = Ax^2+2Ax+2A+Bx^2+Bx+Cx+C$

$x^2: 5 = A+B$

$x: 6 = 2A+B+C \rightarrow 6 = 2A+B+(3-2A) \rightarrow 6 = B+3$

$\#: 3 = 2A+C$

$C = 3-2A$

$B = 3$

$\therefore A = 2$

$\therefore C = -1$

$\therefore \dfrac{2}{x+1} + \dfrac{3x-1}{x^2+2x+2}$

ex 3) $\dfrac{x+4}{x^2(x^2+4)}$ — $x \cdot x \cdot (x^2+4)$ (repeated, prime quad)

set-up only $= \underbrace{\dfrac{A}{x} + \dfrac{B}{x^2}}_{\text{repeated}} + \dfrac{Cx+D}{x^2+4}$

ex 4) $\dfrac{6x^2-x+1}{x^3+x^2+x+1}$ — Factor by grouping.. $x^2(x+1)+1(x+1)$

$= \dfrac{6x^2-x+1}{(x+1)(x^2+1)} = \underbrace{\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+1}}_{\text{set-up only}}$

ex 5) $\dfrac{x^3+x^2+2}{(x^2+2)^2}$ — $(x^2+2)^2$ (prime quad, repeated)

$= \dfrac{Ax+B}{x^2+2} + \dfrac{Cx+D}{(x^2+2)^2}$ *Mult by LCD

FOIL $x^3+x^2+2 = (Ax+B)(x^2+2)+Cx+D$

$x^3+x^2+2 = Ax^3+2Ax+Bx^2+2B+Cx+D$

Ox

$x^3: 1 = A$

$x^2: 1 = B$

$x: 0 = 2A+C \therefore C=-2$

$\#: 2 = 2B+D \therefore D=0$

$\therefore \dfrac{1x+1}{x^2+2} + \dfrac{-2x}{(x^2+2)^2}$

HW: p766, #30, 32, 36, 38