4/4 THU

7.4 Solving Non-Linear Equations (2 variables: Substitution)

ex 1) $\begin{cases} x^2 = y-1 \\ 4x - y = -1 \end{cases}$

* isolate what they have in common....

$x^2 + 1 = y$

$4x - (x^2+1) = -1$

$4x - x^2 - 1 = -1$

$-x^2 + 4x = 0$ ÷ -1

$x^2 - 4x = 0$ * factor

$x(x-4) = 0$

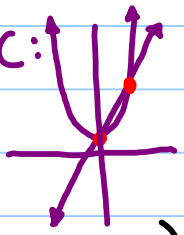
$x = 0 \mid x = 4$

* Subst & Solve for y.

$x: 0 \rightarrow 0^2 + 1 = y;\ 1 = y \rightarrow (0,1)$

$x: 4 \rightarrow 4^2 + 1 = y;\ 17 = y \rightarrow (4,17)$

GC:



ex 2) $\begin{cases} x + y = 6 \\ y = x^2 - 8x + 16 \end{cases}$ → $x + (x^2 - 8x + 16) = 6$

$x^2 - 7x + 10 = 0$

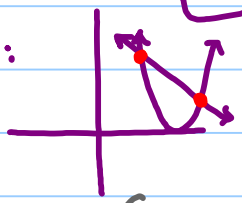
$(x-5)(x-2) = 0$

$x = 5 \mid x = 2$

$y = 1 \quad y = 4$

$\therefore (5,1)$ & $(2,4)$

GC:



ex 3) $\begin{cases} y = x - 3 \\ y^2 = -12x \end{cases}$ → $(x-3)^2 = -12x \rightarrow x^2 - 6x + 9 = -12x \rightarrow x^2 + 6x + 9 = 0$

$\rightarrow (x+3)(x+3) = 0 \rightarrow x = -3$: $y = (-3) - 3 = -6 \therefore (-3,-6)$

-OR- $y + 3 = x$

$y^2 = -12(y+3)$

$y^2 = -12y - 36 \rightarrow y^2 + 12y + 36 = 0 \rightarrow (y+6)^2 = 0 \rightarrow y = -6$;

ex 4) $\begin{cases} xy = 12 \rightarrow y = \frac{12}{x} \\ x^2 + y^2 = 40 \rightarrow x^2 + \left(\frac{12}{x}\right)^2 = 40 \rightarrow \left[x^2 + \frac{144}{x^2} = 40\right] \cdot x^2 \text{ LCD} \end{cases}$

$\rightarrow x^4 + 144 = 40x^2 \rightarrow x^4 - 40x^2 + 144 = 0 \rightarrow (x^2 - 36)(x^2 - 4) = 0$

$(x-6)(x+6)(x-2)(x+2) = 0$

$x = 6, -6, 2, -2 \rightarrow (6,2), (-6,-2), (2,6), (-2,-6)$

ex 5) $\begin{cases} x+2y=0 \\ (x-1)^2+(y-1)^2=5 \end{cases}$

$x+2y=0 \rightarrow x=-2y$

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$(-2y-1)^2+(y-1)^2=5$

$4y^2+4y+1+y^2-2y+1=5$

$5y^2+2y-3=0 \rightarrow (5y-3)(y+1)=0$

$y=\frac{3}{5} \quad | \quad y=-1$

$\left(-\frac{6}{5}, \frac{3}{5}\right)$ & $(2,-1)$

ex 6) $\begin{cases} x+y=-5 \\ x^2+y^2=10y+75 \end{cases}$

$x+y=-5 \rightarrow x=-y-5$

$(-y-5)^2+y^2=10y+75$

$y^2+10y+25+y^2-10y-75=0 \rightarrow 2y^2-50=0 \rightarrow y^2-25=0$

$(y-5)(y+5)=0 \rightarrow y=5,-5$

$(-10,5)$ & $(0,-5)$

HW: p 774, #2–18 even