4/30 TUE

9.1 & 9.2 ....Complete the Square (Ellipses & Hyperbolas)

→ quadratics → perfect square trinomial

ax^2+bx+c ex's: $\underbrace{x^2+2x+1}_{(x+1)^2}$, $\underbrace{x^2+4x+4}_{(x+2)^2}$, $\underbrace{x^2+6x+9}_{(x+3)^2}$...

$Ax^2+By^2+Cx+Dy+\underbrace{E}_{\#}+\ldots$ → standard form: ellipse, hyperbola

Complete the Square

x^2+bx

ex 1) $x^2-10x+25=\#+25$

① $a=1$ ② $\left(\frac{b}{2}\right)^2$ $\left(\frac{-10}{2}\right)^2$

PST → $(x-5)(x-5)$ or $(x-5)^2=\#+25$

ex 2) $x^2+20x+100=1+100$

$a=1$ $\left(\frac{20}{2}\right)^2$

PST

$(x+10)(x+10)=101$

$(x+10)^2=101$

$f(x)=x^2+20x-1$

$=x^2+20x \quad -1$

$\left(\frac{20}{2}\right)^2$

$f(x)=\underbrace{x^2+20x+100}_{PST}-100-1$

$f(x)=(x+10)(x+10)-101$

$f(x)=\underbrace{(x+10)^2-101}_{a(x-h)^2+k}$

ex 3) $4x^2-24x \quad =1$

① $a\neq 1$ → ÷ by "a" (leading coef)

$4(x^2-6x+9)=1+36$

$\left(\frac{-6}{2}\right)^2$ $4\cdot 9$

$4(x-3)(x-3)=37$

$4(x-3)^2=37$

⋮

ex 4) $4x^2+36y^2+16x-72y-92=0$

• group the variables

move the const

$4x^2+16x \quad +36y^2-72y \quad =92$

• coefs of x^2 & y^2 = pos → ellipse

$4(x^2+4x+4)+36(y^2-2y+1)=92+4\cdot 4+36\cdot 1$

$\left(\frac{4}{2}\right)^2 \qquad \left(\frac{-2}{2}\right)^2$

$\frac{4(x+2)^2}{144}+\frac{36(y-1)^2}{144}=\frac{144}{144}$ ← need "1"

$\frac{(x+2)^2}{36}+\frac{(y-1)^2}{4}=1$ (h, k)

$c: (-2,1)$

foci? c?

$a^2-b^2=c^2$

$36-4=c^2$

$32=c^2$

$\pm 4\sqrt{2}=c$

$f: (-2\pm 4\sqrt{2}, 1)$

ex5) $9y^2-16x^2+18y+64x-199=0$

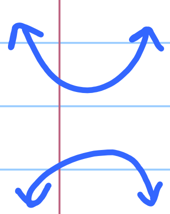
hyperbola

$9y^2+18y \quad -16x^2+64x \quad =199$

$9(y^2+2y+1)-16(x^2-4x+4)=199+9\cdot 1-16\cdot 4$

$\left(\frac{2}{2}\right)^2 \qquad \left(\frac{-4}{2}\right)^2$

$\frac{9(y+1)^2}{144}-\frac{16(x-2)^2}{144}=\frac{144}{144}$



$\frac{(y+1)^2}{16}-\frac{(x-2)^2}{9}=1$ (k, h)

$c: (2,-1)$

foci? c?

$a^2+b^2=c^2$

$16+9=c^2$

$25=c^2$

$c=5$

$f: (2,-1\pm 5)$

$\to (2,4)$ & $(2,-6)$