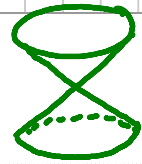
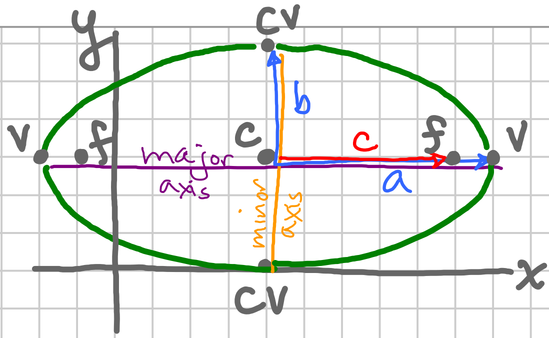
4/25 THU

Ch 9 ~ Conic Sections

9.1 Ellipse



circle, ellipse, parabola, hyperbola, line, point



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

* major axis (vertex to vertex) : $2a$

* minor axis (covertex to covertex) : $2b$

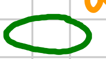
center: (h,k)

center to focus: c ~ foci

$a^2 - b^2 = c^2$

* $a^2 > b^2$

ex 1) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ $(x-0)^2$, $(y-0)^2$

 a^2 ← 25 bigger; 9 ← b^2

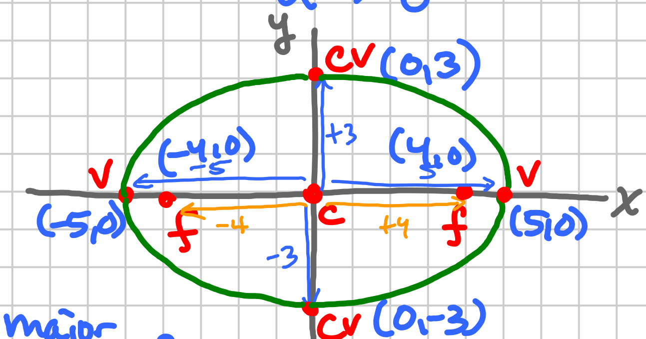
$c: (h,k) \to (0,0)$

$v: (h \pm a, k) \to (\pm 5, 0)$

$cv: (h, k \pm b) \to (0, \pm 3)$

foci?: c? $a^2 - b^2 = c^2$
$25 - 9 = c^2$
$16 = c^2$
$c = 4$

$= (h \pm c, k) = (\pm 4, 0)$



major axis $= 2a = 2(5) = 10$

minor axis $= 2b = 2(3) = 6$

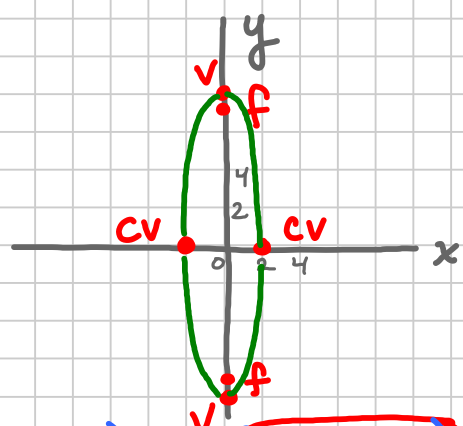
ex 2) $\frac{x^2}{4} + \frac{y^2}{64} = 1$ b^2 — 4, 64 — a^2

center: $(h,k) = (0,0)$

vertices: $(h, k \pm a) = (0, \pm 8)$
major axis : $2a = 2(8) = 16$

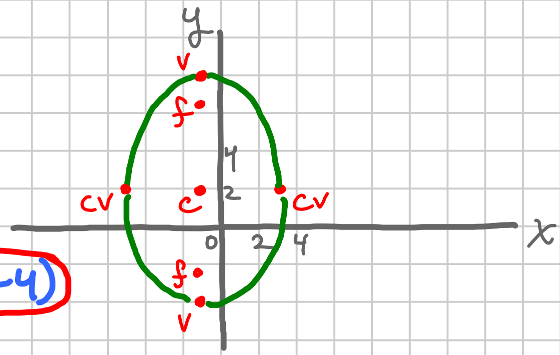
Covertices : $(h \pm b, k) = (\pm 2, 0)$
minor axis : $2b = 2(2) = 4$

foci? c? : $a^2 - b^2 = c^2 \to (h, k \pm c) \to (0, \pm 2\sqrt{15})$
$64 - 4 = c^2$
$60 = c^2$ $c = 2\sqrt{15} \approx 7.75\ldots$



ex 3) $\dfrac{(x+1)^2}{16} + \dfrac{(y-2)^2}{36} = 1$

$16 \leftarrow b^2$ $\quad 36 \leftarrow a^2$



c: $(h,k) = (-1,2)$

v: $(h, k \pm a) = (-1, 2 \pm 6) = (-1,8), (-1,-4)$

major axis $= 2 \cdot a = 2 \cdot 6 = 12$

cv: $(h \pm b, k) = (-1 \pm 4, 2) = (3,2), (-5,2)$

minor axis $= 2b = 2(4) = 8$

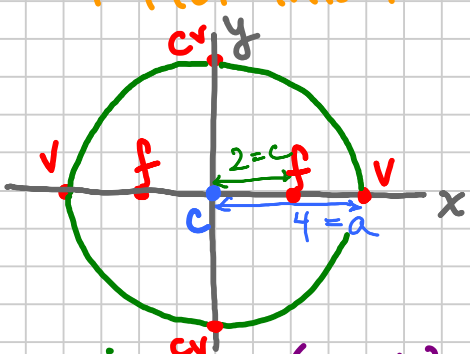
f: $(h, k \pm c)$ $\quad$ c? $\quad a^2 - b^2 = c^2 = (-1, 2 \pm 2\sqrt{5})$

$36 - 16 = c^2$

$20 = c^2$ $\quad c = 2\sqrt{5} \approx 4.47$

ex 4) Given: foci $\rightarrow (2,0), (-2,0)$ & vertices $\rightarrow (-4,0), (4,0)$.
Find everything else (& graph).

* Plot info!



- wider horizontally
- center is halfway the vertices $\rightarrow$ $\overset{h,k}{(0,0)}$
- center to focus: 2 units = "c"
- b? $\quad a^2 - b^2 = c^2$ $\quad b^2 = 12$

$4^2 - b^2 = 2^2$ $\quad b = 2\sqrt{3} \approx 3.46$

$-b^2 = -12$

- cv: $(0, 0 \pm 2\sqrt{3}) \rightarrow (0, \pm 2\sqrt{3})$

major axis $= 2a = 2(4) = 8$

minor axis $= 2b = 2 \cdot 2\sqrt{3} = 4\sqrt{3}$

$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1 \rightarrow \boxed{\dfrac{x^2}{16} + \dfrac{y^2}{12} = 1}$

ex 5) Given $\begin{cases} \text{major axis} = 18 \text{ units (vertical)}. \\ \text{minor axis} = 12 \text{ units} \\ \text{center: } (1,3) \end{cases}$ Find the eqn of the ellipse

$2a \ldots a = 9$

$2b \ldots b = 6$

h, k

$\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1 \Rightarrow \dfrac{(x-1)^2}{36} + \dfrac{(y-3)^2}{81} = 1$

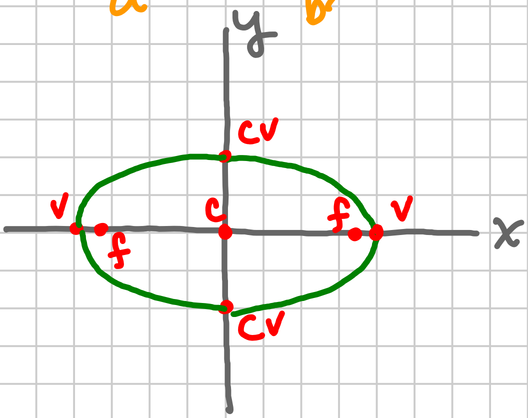
ex 6) $4x^2 = 64 - 16y^2$. Do everything.

$$\begin{aligned} 4x^2 &= 64 - 16y^2 \\ +16y^2 & \qquad +16y^2 \end{aligned}$$

$$\frac{4x^2}{64} + \frac{16y^2}{64} = \frac{64}{64}$$ ← need "1"

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

$a^2 \to 16$ $b^2 \to 4$



Center: $(h,k) \to (0,0)$

vertices: $(h \pm a, k) \to (\pm 4, 0)$

major axis: $2a = 2(4) = 8$

covertices: $(h, k \pm b) \to (0, \pm 2)$

minor axis: $2b = 2(2) = 4$

foci: $(h \pm c, k)$ $\to (\pm 2\sqrt{3}, 0)$

c? $a^2 - b^2 = c^2$

$16 - 4 = c^2$

$12 = c^2$

$2\sqrt{3} = c$

$c \approx 3.5$