Trigonometry / Pre-Calculus
CRT
Formula Sheet

The formulas below may be helpful in answering some of the questions on the Trigonometry / Pre-Calculus Assessment.

Trigonometry Formulas/Identities

1. Sum and Difference
   a) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
   b) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
   c) $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$
2. Double Angle
   a) $\sin 2\theta = 2\sin\theta\cos\theta$
   b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
   c) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
3. Half Angle
   a) $\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$
   b) $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$
4. Pythagorean Identities
   a) $\cos^2\theta + \sin^2\theta = 1$ $\quad 1+\tan^2\theta = \sec^2\theta$ $\quad \cot^2\theta + 1 = \csc^2\theta$
5. a) Law of Sines $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$
   b) Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos A$
6. Area of Triangles
   a) $A = \dfrac{1}{2}ab\sin C$
   b) $A = \sqrt{s(s-a)(s-b)(s-c)},\ s = \dfrac{1}{2}(a+b+c)$
7. Arc Length $S = r\theta$

   Area of Sector $A = \dfrac{1}{2}r^2\theta$

Inner Product of Vectors in a Plane

$$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2$$

Inner Product of Vectors in Space

$$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

Cross Product of Vectors in Space

$$\vec{a}\times\vec{b} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\vec{k}$$

Complex Numbers

1. Products $r_1(\cos\alpha + i\sin\alpha)\cdot r_2(\cos\beta + i\sin\beta) = r_1r_2[\cos(\alpha+\beta) + i\sin(\alpha+\beta)]$

   Quotients $\dfrac{r_1(\cos\alpha + i\sin\alpha)}{r_2(\cos\beta + i\sin\beta)} = \dfrac{r_1}{r_2}[\cos(\alpha-\beta) + i\sin(\alpha-\beta)]$

   Powers $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

   Roots $[r(\cos\theta + i\sin\theta)]^{\frac{1}{n}} = r^{\frac{1}{n}}(\cos\dfrac{\theta}{n} + i\sin\dfrac{\theta}{n})$

Converting Rectangular Coordinates to Polar Coordinates

$r = \sqrt{x^2 + y^2}$

$\theta =$ Arctan $\dfrac{y}{x}$, when $x > 0$

$\theta =$ Arctan $\dfrac{y}{x} + \pi$, when $x < 0$

Converting Polar Coordinates to Rectangular Coordinates

$x = r\cos\theta$

$y = r\sin\theta$

Power-Reducing Formula

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$\tan^2\theta = \dfrac{1-\cos 2\theta}{1+\cos 2\theta}$

Product-to-sum Formula

$\sin\alpha\sin\beta = \dfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\cos\alpha\sin\beta = \dfrac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$

Sum-to-product Formula

$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\sin\dfrac{\alpha-\beta}{2}\cos\dfrac{\alpha+\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

Roots

$z_k = \sqrt[n]{r}\left[\cos\left(\dfrac{\theta+2\pi k}{n}\right) + i\sin\left(\dfrac{\theta+2\pi k}{n}\right)\right]$