Set A: Nonfractional Equations

In 1 – 14, prove that the equation is an identity.

1. $\tan x \csc x = \sec x$
2. $\cos x \csc x = \cot x$
3. $\cos^2 x = \csc x \sin x - \sin^2 x$
4. $\tan^2 x = \sec^2 x - \cos x \sec x$
5. $\tan^2 x + \sin^2 x + \cos^2 x = \sec^2 x$
6. $\cot^2 x = \csc^2 x - \sin^2 x - \cos^2 x$
7. $\tan^2 x = (\sec x - 1)(\sec x + 1)$
8. $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$
9. $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$
10. $(\sin\theta - \cos\theta)^2 = 1 - 2\sin\theta\cos\theta$
11. $\tan^2 A(1 + \cot^2 A) = \sec^2 A$
12. $(1 - \sin^2\theta)(\sec^2\theta - 1) = \sin^2\theta$
13. $\sin^2 x + \sin^2 x \tan^2 x = \tan^2 x$
14. $(1 + \sec x)(1 - \cos x) = \cos x \tan^2 x$

Set B: Fractional Equations

In 15 – 35, prove that the equation is an identity.

15. $\tan x \cos x = \dfrac{1}{\csc x}$
16. $\sin x \cot x = \dfrac{1}{\sec x}$
17. $\sin\theta \sec\theta = \dfrac{1}{\cot\theta}$
18. $\cos x = \dfrac{\cot x}{\csc x}$
19. $\cot x = \dfrac{\csc x}{\sec x}$
20. $\dfrac{\sec x}{\tan x} = \dfrac{\cot x}{\cos x}$
21. $\dfrac{\sin x \csc x}{\tan x} = \cot x$
22. $\dfrac{\tan x}{\sec x} = \dfrac{\cos x}{\cot x}$
23. $\dfrac{\sin x \csc x}{\cos x} = \sec x$
24. $\cos\theta \csc\theta = \dfrac{1}{\tan\theta}$
25. $(1 - \cos x)(1 + \cos x) = \dfrac{1}{\csc^2 x}$
26. $\dfrac{(1 + \sin x)^2}{\cos^2 x} = \dfrac{1 + \sin x}{1 - \sin x}$
27. $\dfrac{1 - \cos A}{\sin A} = \csc A - \cot A$
28. $\dfrac{1 + \tan A}{\sin A} = \csc A + \sec A$
29. $\dfrac{1 + \sec x}{\csc x} = \sin x + \tan x$
30. $\dfrac{\sec A}{\cot A + \tan A} = \sin A$
31. $\dfrac{\sin x}{1 + \cos x} + \cot x = \csc x$
32. $\dfrac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$
33. $\dfrac{\cot A}{\tan A} + \dfrac{\tan A}{\cot A} = \dfrac{\cot^4 A + 1}{\cot^2 A}$
34. $\dfrac{\cos\theta \sin^2\theta}{1 + \cos\theta} = \cos\theta - \cos^2\theta$
35. $\dfrac{\sin\theta}{1 + \cos\theta} + \dfrac{1 + \cos\theta}{\sin\theta} = 2\cot\theta\sec\theta$

Set C: Generalized Trigonometric Relationships
(Double Angle, Half Angle, Sums and Differences of Angles, etc.)

In 36 – 59, prove that the equation is an identity.

36. $(\sin x - \cos x)^2 = 1 - \sin 2x$

37. $\tan A \sin 2A = 2\sin^2 A$

38. $\sin 2x = \dfrac{2\tan x}{1+\tan^2 x}$

39. $\cos 2A = \dfrac{2-\sec^2 A}{\sec^2 A}$

40. $\csc 2x = \dfrac{\sec x}{2\sin x}$

41. $\tan 2\theta = \dfrac{2\tan\theta}{\sec^2\theta - 2\tan^2\theta}$

42. $\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x} = \dfrac{2}{\sin 2x}$

43. $\sec 2x = \dfrac{1+\tan^2 x}{1-\tan^2 x}$

44. $\dfrac{1-\cos 2x}{\sec^2 x - \tan^2 x} = 2\sin^2 x$

45. $\cot\theta - \dfrac{\cos 2\theta}{\sin\theta\cos\theta} = \tan\theta$

46. $\dfrac{\cos B + \sin B}{\cos B - \sin B} = \dfrac{\sin 2B + 1}{\cos 2B}$

47. $\dfrac{\cos 2x}{\sin x} + \dfrac{\sin 2x}{\cos x} = \csc x$

48. $\dfrac{\sin 2x}{\sin x} - \dfrac{\cos 2x}{\cos x} = \sec x$

49. $\dfrac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$

50. $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{\tan x + \tan y}{\tan x - \tan y}$

51. $\tan x = \dfrac{\sin(x-y)}{\cos x \cos y} + \tan y$

52. $\tan\left(\dfrac{\pi}{4} + x\right) = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

53. $\tan(45° + A) = \dfrac{\cos A + \sin A}{\cos A - \sin A}$

54. $\tan(45° + x) = \dfrac{\cos 2x}{1 - \sin 2x}$

55. $\dfrac{\sin(x+y) + \sin(x-y)}{\sin x} = 2\cos y$

56. $\sin^2 \dfrac{x}{2} = \dfrac{\sec x - 1}{2\sec x}$

57. $\tan\dfrac{1}{2}x = \dfrac{\sin x}{1+\cos x}$

58. $\cot\dfrac{1}{2}x = \csc x + \cot x$

59. $\sec^2 \dfrac{1}{2}x = \dfrac{2(1-\cos x)}{\sin^2 x}$