

1-3

Solving Equations With a Variable on Both Sides

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EXPLORE & REASON

Some friends want to see a movie that is showing at two different theaters in town. They plan to share 3 tubs of popcorn during the movie.



	Theater A	Theater B
Ticket Price	\$14.50	\$13.00
Popcorn	\$5.75	\$6.75
	$\times 3$	$\times 3$
	17.25	20.25

A. Construct Arguments Which movie theater should the friends choose? Explain. © MP3

A	17.25	46.25
B	20.25	46.25

Handwritten notes on the table:
 - A red circle highlights the total cost for Theater B (46.25).
 - An orange circle highlights the total cost for Theater A (46.25).
 - Green arrows show calculations: Theater A popcorn is \$5.75 x 3 = \$17.25; Theater B popcorn is \$6.75 x 3 = \$20.25. Theater A tickets are \$14.50 x 2 = \$29. Theater B tickets are \$13.00 x 2 = \$26.
 - Red text on the right says: "With 3 or more people, Theater B is cheaper".

B. For what situation would the total cost at each theater be exactly the same? Explain.

two friends w/ 3 tubs of popcorn
\$46.25

C. There are different methods to solving this problem. Which do you think is the best? Why?

Table, spreadsheet, ...
manually add...

HABITS OF MIND

Make Sense and Persevere What assumptions did you make that helped you work through the Explore & Reason? © MP.1

EXAMPLE 1

Try It! Solve Equations With a Variable on Both Sides

- Combine like terms
 - Undo operations
- PE
MD
AS

ex 1) $3x - 10 + 4x = -2(x - 4) + 9$

Distributive Prop

$$7x - 10 = -2(x - 4) + 9$$

$$7x - 10 = -2x + 8 + 9$$

$$7x - 10 = -2x + 17$$

Undo operations

$$\begin{array}{r} 7x - 10 \\ + 2x \\ \hline 9x - 10 \end{array} = \begin{array}{r} -2x + 17 \\ + 2x \\ \hline 17 \end{array}$$

$$\begin{array}{r} 9x - 10 \\ + 10 \\ \hline 9x \end{array} = \begin{array}{r} 17 \\ + 10 \\ \hline 27 \end{array}$$

$$\frac{9x}{9} = \frac{27}{9}$$

$$x = 3$$

1. Solve each equation.

a. $100(z - 0.2) = -10(5z + 0.8)$

$$\begin{array}{r} 100z - 20 \\ + 50z \\ \hline 150z - 20 \end{array} = \begin{array}{r} -50z - 8 \\ + 50z \\ \hline -8 \end{array}$$

$$\begin{array}{r} 150z - 20 \\ + 20 \\ \hline 150z \end{array} = \begin{array}{r} -8 \\ + 20 \\ \hline 12 \end{array}$$

$$\frac{150z}{150} = \frac{12}{150}$$

$$z = \frac{6}{75} = \frac{2}{25}$$

b. $\frac{5}{8}(16d + 24) = 6(d - 1) + 1$

$$\begin{array}{r} 5 \cdot 16d + 5 \cdot 24 \\ \hline 8 \end{array} = \begin{array}{r} 6d - 6 + 1 \\ \hline \end{array}$$

$$\begin{array}{r} 10d + 15 \\ - 6d \\ \hline 4d + 15 \end{array} = \begin{array}{r} 6d - 5 \\ - 6d \\ \hline -5 \end{array}$$

$$\begin{array}{r} 4d + 15 \\ - 15 \\ \hline 4d \end{array} = \begin{array}{r} -5 \\ -15 \\ \hline -20 \end{array}$$

$$\frac{4d}{4} = \frac{-20}{4}$$

$$d = -5$$

EXAMPLE 2

Try It! Understand Equations with Infinitely Many or No Solutions

* variables disappear
???

2. Solve each equation. Is the equation an identity? Explain.

a. $t - 27 = -(27 - t)$

$$\begin{array}{r} t - 27 \\ - t \\ \hline -27 \end{array} = \begin{array}{r} -27 + t \\ - t \\ \hline -27 \end{array}$$

TRUE Statement

→ ∞ solutions or all real #s

Identity

b. $16(4 - 3m) = 96(-\frac{m}{2} + 1)$

$$\begin{array}{r} 64 - 48m \\ + 48m \\ \hline 64 \end{array} = \begin{array}{r} -48m + 96 \\ + 48m \\ \hline 96 \end{array}$$

FALSE Statement

→ no solution!

HABITS OF MIND

Construct Arguments One student maintains that the order in which terms are collected on each side of an equation does not matter. Construct an argument to support or refute the student's position. © MP.3



EXAMPLE 3 Try It! Solve Mixture Problems

3. How many pounds of Arabica coffee should you mix with 3 pounds of Robusta coffee to make a coffee blend that costs \$12.50 per pound?

Coffee	\$/lb	lb	Total Cost
Arabica	28	$\cdot a$	$= 28a$
Robusta	8.75	$\cdot 3$	$= 26.25$
Blend	15.50	$\cdot (a+3)$	$= 15.50(a+3)$

15.50 Blend

$$28a + 26.25 = 15.50(a+3)$$

$$28a + 26.25 = 15.50a + 46.5$$

$$\begin{array}{r} -15.50a - 26.25 \\ \hline 12.5a = 20.25 \\ 12.5 \\ \hline a = 1.62 \text{ lbs of Arabica} \end{array}$$

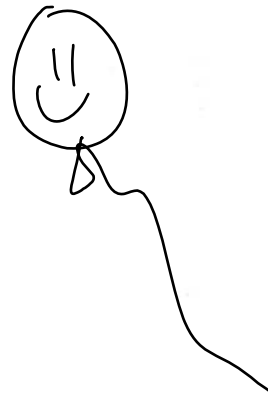
HABITS OF MIND

Generalize How can you determine whether an equation has infinitely many or no solutions?  MP.8

→ when the variable(s) disappear...

EXAMPLE 4 Try It! Use Equations to Solve Problems

4. Cameron's friend tells him of another service that has a \$15 joining fee but charges \$0.80 per song. At what number of songs does this new service become a less expensive option to Cameron's current service?



Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How do you create equations with a variable on both sides and use them to solve problems?

2. **Vocabulary** Why does it make sense to describe an equation that has infinitely many solutions as an *identity*?

3. **Error Analysis** Isabel says that the equation $x - 2 = -(x - 2)$ has no solution because a number can never be equal to its opposite. Explain the error Isabel made. **MP.3**

4. **Look for Relationships** You are solving an equation with a variable on each side. Does the side on which you choose to isolate the variable affect the solution? Why might you choose one side over the other? **MP.7**

Do You KNOW HOW?

Solve each equation.

5. $5(2x + 6) = 8x + 48$
 $10x + 30 = 8x + 48$
 $\begin{array}{r} 10x + 30 \\ -8x \\ \hline 2x + 30 \end{array} = \begin{array}{r} 8x + 48 \\ -8x \\ \hline 48 \end{array}$
 $\begin{array}{r} 2x + 30 \\ -30 \\ \hline 2x \end{array} = \begin{array}{r} 48 \\ -30 \\ \hline 18 \end{array}$
 $\frac{2x}{2} = \frac{18}{2}$ **$x = 9$**

Distrib Prop
Combine like terms
Undo Ops

PE
MD
AS

SOLVE

6. $-3(8 + 3h) = 5h + 4$
 $-24 - 9h = 5h + 4$
 $\begin{array}{r} -24 - 9h \\ +9h \\ \hline -24 \end{array} = \begin{array}{r} 5h + 4 \\ +9h \\ \hline 14h + 4 \end{array}$
 $\begin{array}{r} -24 \\ -4 \\ \hline -28 \end{array} = \begin{array}{r} 14h + 4 \\ -4 \\ \hline 14h \end{array}$
 $\frac{-28}{14} = \frac{14h}{14}$ **$-2 = h$**

7. $2(y - 6) = 3(y - 4) - y$

$2y - 12 = 3y - 12 - y$
 $2y - 12 = 2y - 12$
true statement

∞ solutions identity

8. $8x - 4 = 2(4x - 4)$

$8x - 4 = 8x - 8$
false statement

no solution

9. For how many games is the total cost of bowling equal for the two bowling establishments?

Family Bowling

Cost (dollars)	Game	4.00
	Shoes	1.00

Knight Owl Bowling

Cost (dollars)	Game	3.75
	Shoes	2.00

var: x

$4x + 1 = 3.75x + 2$
 $\begin{array}{r} 4x + 1 \\ -3.75x \\ \hline 0.25x + 1 \end{array} = \begin{array}{r} 3.75x + 2 \\ -3.75x \\ \hline 2 \end{array}$
 $\begin{array}{r} 0.25x + 1 \\ -1 \\ \hline 0.25x \end{array} = \begin{array}{r} 2 \\ -1 \\ \hline 1 \end{array}$
 $\frac{1}{4} \cdot \frac{1}{4}x = \frac{1 \cdot 4}{1}$
 $x = 4$ games

FB	\$1	5	9	13	17
KOB	\$2	5.75	9.5	13.25	17

Shoes +4
3.75