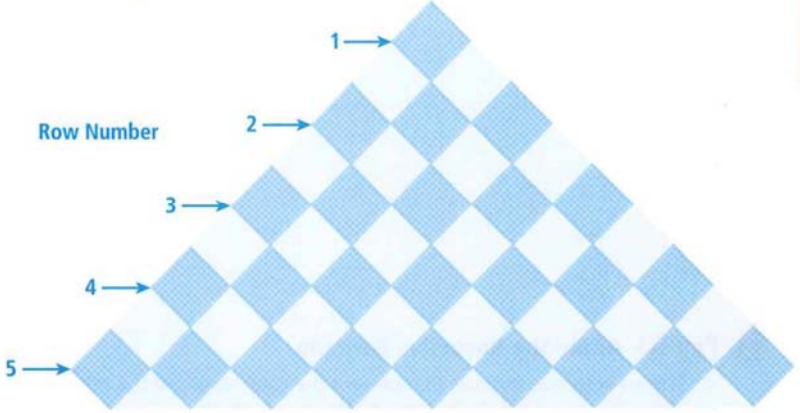


**3-4**  
Arithmetic Sequences  
PearsonRealize.com

**EXPLORE & REASON**

A fashion designer is designing a patterned fabric.



A. Copy and complete.

Row number	1	2	3	4	5
Number of Patterned Squares in the Row	1	3	5	7	9
Total Number of Patterned Squares	1	4	9	16	25

+2 +2 +2 +2 ← d  
1 2 3 4 5  
1<sup>2</sup> 2<sup>2</sup> 3<sup>2</sup> 4<sup>2</sup> 5<sup>2</sup>

Sequence: ordered list of numbers

arithmetic sequence: Common difference between each term  
±d

odd #s  
perfect squares

Recursive Formula: relies on previous terms

ex) 1, 3, 5, 7, 9, ...

$a_1$  ← 1st term

"add 2 to previous term"

$a_n = a_{n-1} + d$  ← arithmetic sequence (d)

n<sup>th</sup> term  
previous term

$a_1 = 1$   
 $a_n = a_{n-1} + 2$

Write a recursive formula for:

ex 1) -3, -1, 1, 3, 5, ...

1st term  
arith?  $a_1 = -3$

$a_n = a_{n-1} + 2$   
n<sup>th</sup> term      previous term

**HABITS OF MIND**

Model with Mathematics What information would you need from the table to write a linear equation that represents the pattern? Explain. © MP.4

ex 2) 100, 89, 78, ...

$a_1 = 100$   
 $a_n = a_{n-1} - 11$

ex) arithmetic sequence

3, 7, 11, 15, ...  
d: 4

$a_1$ : 1st term :  $a(1)$   
Subscript notation  
function notation

Recursive Formula • relies on previous terms --

$a_1 = 3$   
 $a_n = a_{n-1} + 4$   
Common difference  
previous term  
nth term

$a_1 = 3$   
 $a_2 = 3 + 4$   
 $a_3 = 3 + 4 + 4$   
 $a_4 = 3 + 4 + 4 + 4$   
 $(n-1)d$

EXAMPLE 2 Try It! Connect Sequences and Functions

1. Is the domain of the function in Part B of Example 1 continuous or discrete? Explain.

Explicit Formula (arithmetic sequence)

$$a_n = a_1 + (n-1)d$$

nth term    1st term    term #    Common diff

EXAMPLE 2 Try It! Apply the Recursive Formula

2. Write a recursive formula to represent the total height of the nth stair above the ground if the height of each stair is 11 cm.

14, 11, 8, ... d?  $11-14 = -3$

$a_1 = 14$   
 $a_n = a_{n-1} + -3$  Recursive

EXAMPLE 3 Try It! Apply the Explicit Formula

$$a_n = a_1 + d(n-1)$$

3. The cost to rent a bike is \$28 for the first day plus \$3 for each day after that. Write an explicit formula for the rental cost for n days. What is the cost of renting the bike for 8 days?

14, 11, 8, ...  
 $a_1$ , d: -3 explicit

$$a_n = a_1 + (n-1)d$$

$$a_n = 14 + (n-1)(-3)$$

Find the 30th term.  
n

$$a_{30} = 14 + (30-1)(-3)$$

$$= 14 + (29)(-3)$$

$$= 14 - 87 = -73$$

HABITS OF MIND

Reason Can a recursive formula have a negative common difference? Explain. © MP.2

Year

**EXAMPLE 4** Try It! Write an Explicit Formula From a Recursive Formula

4. Write an explicit formula for each arithmetic sequence.

Given:  
Recursive  
Formula

a.  $a_n = a_{n-1} - 3; a_1 = 10$

$$a_n = a_1 + (n-1)d$$

10          -3

$$a_n = 10 + (n-1)(-3)$$

b.  $a_n = a_{n-1} + 2.4; a_1 = -1$

$$a_n = a_1 + (n-1)d$$

-1          2.4

$$a_n = -1 + (n-1)(2.4)$$

**EXAMPLE 5** Try It! Write a Recursive Formula From an Explicit Formula

5. Write a recursive formula for each explicit formula.

Explicit  
formula

a.  $a_n = 8 + 3n$

$$\begin{aligned} a_1 &= 8 + 3(1) \\ &= 11 \\ a_2 &= 8 + 3(2) \\ &= 14 \end{aligned}$$

$$a_1 = 11$$

1st term?

$$a_n = a_{n-1} + 3$$

Common  
diff?

b.  $a_n = 12 - 5n$

$$\begin{aligned} a_1 &= 12 - 5(1) \\ &= 7 \\ a_2 &= 12 - 5(2) \\ &= 2 \end{aligned}$$

$$a_1 = 7$$

$$a_n = a_{n-1} + (-5)$$

**HABITS OF MIND****Communicate Precisely** Explain how you can use the recursive formula to find the value of any term in an arithmetic sequence. © MP.6



**Do You UNDERSTAND?**

1. **ESSENTIAL QUESTION** How are arithmetic sequences related to linear functions?

2. **Error Analysis** A student uses the explicit formula  $a_n = 5 + 3(n - 1)$  for the sequence 3, 8, 13, 18, 23, ... to find the 12th term. Explain the error the student made. © MP.3

3. **Vocabulary** When is a recursive formula more useful than an explicit formula for an arithmetic sequence?

4. **Communicate Precisely** Compare and contrast a recursive formula and an explicit formula for an arithmetic sequence. © MP.6

**Do You KNOW HOW?**

Tell whether or not each sequence is an arithmetic sequence.

5. 15, 13, 11, 9, ...

$d: -2$

→ Arithmetic

6. 4, 7, 10, 14, ...

$+3 +3 +4$

→ not Arithmetic

Write a recursive formula for each sequence.

7. 81, 85, 89, 93, 97, ...

$$\begin{cases} a_1 = 81 \\ a_n = a_{n-1} + 4 \end{cases}$$

*previous term*

8. 47, 39, 31, 23, 15, ...

$$\begin{cases} a_1 = 47 \\ a_n = a_{n-1} - 8 \end{cases}$$

9. An online store charges \$5 to ship one box and \$10 to ship two boxes. Write an explicit formula for an arithmetic sequence to represent the amount the online store pays to ship  $n$  boxes. Use the explicit formula to determine how much the online store charges when shipping

$d: +5$

11 items.

5, 10, 15, 20, 25, ...

$n:1$   $n:2$   $n:3$  ...

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ a_{11} &= 5 + (11-1)(5) \\ &= 5 + (10)(5) \\ &= 5 + 50 = 55 \end{aligned}$$

$$\begin{aligned} &= 5 + (10)(5) \\ &= 5 + 50 = 55 \end{aligned}$$