

5-2

Piecewise-Defined Functions

PearsonRealize.com

EXPLORE & REASON

In a relay race, each runner carries a baton for an equal distance, with each runner handing off the baton to the next runner.

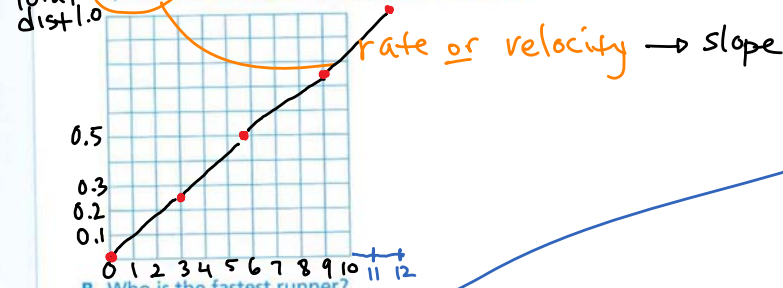
| Path of the Baton | | |
|-------------------|------------|---------------------|
| | Time (min) | Total Distance (mi) |
| Start | 0 | 0 |
| Runner 1 | 3 | 0.25 |
| Runner 2 | 5.75 | 0.50 |
| Runner 3 | 9 | 0.75 |
| Runner 4 | 11.50 | 1.00 |

$$\frac{0.50 - 0.25}{5.75 - 3} = \frac{0.25}{2.75}$$

$$\frac{1 - 0.75}{11.5 - 9} = \frac{0.25}{2.5} = 0.10 \quad \text{fastest}$$

piecewise function

A. Graph the distance traveled by the baton as a function of time. How is the speed of each runner represented in the graph?



B. Who is the fastest runner?

C. **Communicate Precisely** How is the graph of this function similar to the graph of a linear function? How is it different? © MP.6

No

→ Piecewise graph...

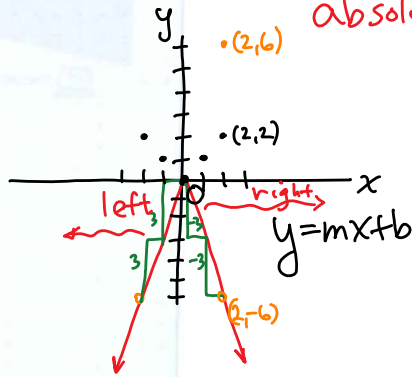
HABITS OF MIND

Use Appropriate Tools How does making a graph help you understand the function and the situation? Explain. © MP.5



EXAMPLE 1 Try It! Understand Piecewise-Defined Functions

1. Express $f(x) = -3|x|$ as a piecewise-defined function.



$a: -3 \rightarrow$ vert reflection & stretch

absolute-value...
V shaped

$$f(x) = \begin{cases} 3x + 0 & x \leq 0 \\ -3x + 0 & x \geq 0 \end{cases}$$

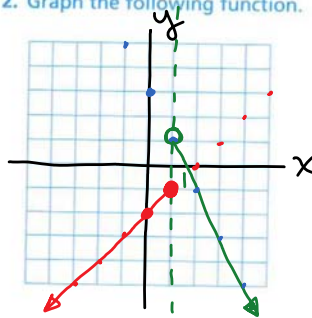
Domain
left of zero
 $x \leq 0$

right of
 $x \geq 0$

EXAMPLE 2 Try It! Graph a Piecewise-Defined Function

2. Graph the following function.

$$f(x) = \begin{cases} x - 2, & x \leq 2 \\ -2x + 3, & x > 2 \end{cases}$$



domain
left less than or equal (closed)
right greater (open) than

HABITS OF MIND

Communicate Precisely How does the domain of the function relate to the piecewise-defined function notation? © MP.6

$d =$ all real numbers...


EXAMPLE 3  **Try It!** Analyze the Graph of a Piecewise-Defined Function

3. Make a conjecture about why a utility company might charge higher rates for greater levels of water consumption.

**EXAMPLE 4**  **Try It!** Apply a Piecewise-Defined Function

4. What is the difference in cost between one order of 200 wristbands, two orders of 100 wristbands each, and four orders of 50 wristbands each?

**HABITS OF MIND**

Look for Relationships How can you tell when the pieces of a piecewise-defined function do not connect? Explain.  MP.7



Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** What are the key features of piecewise-defined functions?

2. **Construct Arguments** If the domain of a piecewise-defined function f is all real numbers, must the range of f also be all real numbers? Explain. **MP.3**

3. **Error Analysis** Liz wrote the following piecewise-defined function:

$$f(x) = \begin{cases} x - 3, & x \leq -3 \\ -2x - 4, & x \geq -3 \end{cases}$$

What is the error that Liz made? **MP.3**

4. **Reason** How many pieces does the absolute value function have? Explain. **MP.2**

Do You KNOW HOW?

Express each function as a piecewise-defined function.

5. $f(x) = 5|x|$

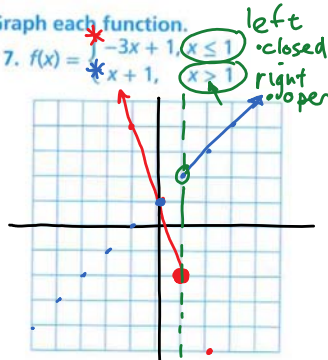
$$f(x) = \begin{cases} -5x & x \leq 0 \\ 5x & x \geq 0 \end{cases}$$

6. $f(x) = -2|x|$

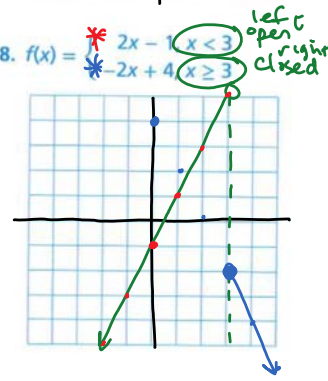
$$f(x) = \begin{cases} 2x & x \leq 0 \\ -2x & x \geq 0 \end{cases}$$

Graph each function.

7. $f(x) = \begin{cases} -3x + 1, & x \leq 1 \\ x + 1, & x > 1 \end{cases}$



8. $f(x) = \begin{cases} 2x - 1, & x < 3 \\ -2x + 4, & x \geq 3 \end{cases}$



9. A function f is defined by the rule $-0.5x + 1$ for the domain $x < 1$ and by the rule x for the domain $x \geq 1$. Write the piecewise-defined function using function notation.