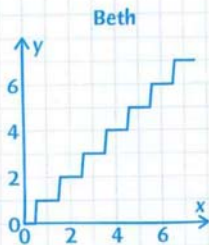


**CRITIQUE & EXPLAIN**

Students are told there is a function where decimals are the inputs and each decimal is rounded to the nearest whole number to get the output. Beth and Latoya each make a sketch of the graph of the function.



**A. Make Sense and Persevere** What is causing both students to create graphs that look like steps? **MP.1**

x	y
0	0
0.1	0
0.2	0
0.5	1
0.6	1
⋮	⋮

Rounding to the nearest whole number...

**B.** Which graph do you think is correct? Explain.

Latoya...

**C.** What does the graph of this function look like? Explain.

Steps ... w/ closed & open endpoints...

**HABITS OF MIND**

**Communicate Precisely** In the graphs, what information do the pairs of open and closed circles convey that the connected lines do not? **MP.6**

Included

not included

round up (integer)

EXAMPLE 1

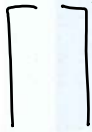


Try It! Understand Step Functions

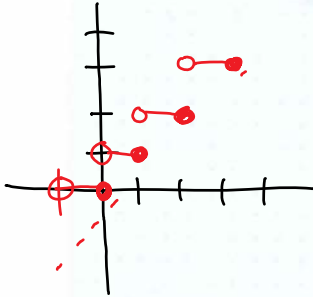
• piecewise function w/ constant pieces



Assess



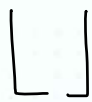
→ Ceiling function



1. Evaluate each function for the given value.

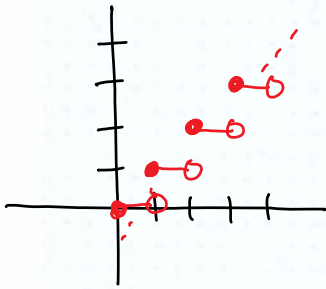
a.  $f(x) = \lceil x \rceil$ ;  $x = 2.65$

$$f(2.65) = \lceil 2.65 \rceil = 3$$



→ Floor function

round down (integer)



b.  $f(x) = \text{floor}(x)$ ;  $x = 2.19$

$$f(2.19) = \text{floor}(2.19) = 2$$

**HABITS OF MIND**

**Look for Relationships** Why are the graphs of floor and ceiling functions composed of horizontal sections? © MP.7




**EXAMPLE 2**  **Try It!** Use a Step Function to Represent a Real-World Situation

2. The postage for a first-class letter weighing one ounce or less is \$0.47. Each additional ounce is \$0.21. The maximum weight of a first-class letter is  $3\frac{1}{2}$  oz. Write a function to represent the situation.

**EXAMPLE 3**  **Try It!** Use a Step Function

3. You rent a karaoke machine at 1 P.M. and plan to return it by 4 P.M. Will you save any money if you return the machine 15 min early? Explain.

**HABITS OF MIND**

**Model With Mathematics** Why are ceiling functions useful in modeling real-world situations?  MP.4



**Do You UNDERSTAND?**

1. **ESSENTIAL QUESTION** How are step functions related to piecewise-defined functions?

2. **Vocabulary** How are the *ceiling function* and the *floor function* similar? How are they different?

3. **Error Analysis** Jason defined the following step function.

$$f(x) = \begin{cases} 5, & 0 \leq x \leq 10 \\ 6, & 10 \leq x \leq 20 \\ 7, & 20 \leq x \leq 30 \end{cases}$$

What is the error that Jason made? © MP.3

4. **Reason** For the function that rounds numbers to the nearest whole number, what are the pieces of the domain for the interval from 0 to 4? © MP.2

**Do You KNOW HOW?**

Evaluate the ceiling function for the given value.

5.  $f(x) = \lceil x \rceil$ ;  $x = 5.13$

$$f(5.13) = \lceil 5.13 \rceil = 6$$

6.  $f(x) = \text{ceiling}(x)$ ;  $x = 11.71$

$$f(11.71) = \text{ceiling}(11.71) = 12$$

Evaluate the floor function for the given value.

7.  $f(x) = \lfloor x \rfloor$ ;  $x = 9.37$

$$f(9.37) = \lfloor 9.37 \rfloor = 9$$

8.  $f(x) = \text{floor}(x)$ ;  $x = 5.49$

$$f(5.49) = \text{floor}(5.49) = 5$$

9. Graph the function  $f$ .

$x$	$f(x)$
$0 < x \leq 1$	4
$1 < x \leq 2$	5
$2 < x \leq 3$	6
$3 < x \leq 4$	7
$4 < x \leq 5$	8
$5 < x \leq 6$	9

