

Squares  $\{1^2, 2^2, 3^2, 4^2, \dots$   
 $\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, \dots$

Cubes  $\{1^3, 2^3, 3^3, 4^3, \dots$   
 $\{1, 8, 27, 64, 125, 216, \dots$

add exponents  $x^3 \cdot x^2 \rightarrow x^{3+2} = x^5$

subtract exponents  $\frac{x^3}{x^2} \rightarrow x^{3-2} = x^1 = x$

multiply exponents  $(x^3)^2 \rightarrow x^{3 \cdot 2} = x^6$

**6-1**  
 Rational Exponents and Properties of Exponents

**CRITIQUE & EXPLAIN**

Students are asked to write an equivalent expression for  $3^{-3}$ . Casey and Jacinta each write an expression on the board.

Casey  
 $3^{-3} = -27$

Jacinta  
 $3^{-3} = \frac{1}{27}$

$3^{-3} \rightarrow \frac{1}{3^3} \rightarrow \frac{1}{27}$

A. Who is correct, Casey or Jacinta? Explain.

negative exponents  $\neq$  negative answer

ex)  $\frac{5^{-2}}{1} = \frac{1}{5^2} = \frac{1}{25}$

$a^{-m} = \frac{1}{a^m}$   
 reciprocate the base

$a^0 = 1$   
 zero power

$\frac{x^3}{x^5} \rightarrow x^{3-5} = x^{-2} = \frac{1}{x^2}$

$x^{-2} \rightarrow \frac{1}{x^2}$

B. Reason. What is the most likely error that was made? © MP.2

$3^2 \rightarrow 3 \cdot 3 \rightarrow 9$

"square root"  
 $\sqrt{9} \rightarrow \sqrt{3 \cdot 3} \rightarrow 3$

**HABITS OF MIND**

Look for Relationships How do you know when exponential expressions are equivalent? Explain. © MP.7

$\begin{cases} x+1 = 4 \\ x = 3 \end{cases}$

$5^{x+1} = 5^4$   
 same

when the bases are the same...

If  $a^m = a^n$  then  $m = n$

# Rational Exponents (fraction powers)

$a^{\frac{m}{n}}$ 
power
radical form
Notes
 $\rightarrow$ 
 $\sqrt[n]{a^m}$ 
nth root of a to m power...

ex)  $6^{\frac{3}{4}}$ 
rati'l exps
fraction
 $\rightarrow$ 
 $\sqrt[4]{6^3}$ 
4th root of 6 to the 3rd power...
  
or
 $(\sqrt[4]{6})^3$ 
radical

## EXAMPLE 1 Try It! Write Radicals Using Rational Exponents

1. How can you write  $\sqrt[3]{2}$  using rational exponents?

$2^{\frac{1}{3}}$ 
↔
 $\sqrt[3]{2^1}$ 
cube root of 2.
  
 $\approx 1.2599$

## EXAMPLE 2 Try It! Use the Product of Powers Property to Solve Equations With Rational Exponents

2. What is the solution of  $(2^{\frac{x}{4}})(2^{\frac{x}{6}}) = 2^3$ ?

$2^{\frac{x}{4}} \cdot 2^{\frac{x}{6}} = 2^3$ 
add exponents
  
 $2^{\frac{x}{4} + \frac{x}{6}} = 2^3$ 
  
 $2^{\frac{3x}{12} + \frac{2x}{12}} = 2^3$ 
  
 $2^{\frac{5x}{12}} = 2^3$ 
  
 $\frac{5x}{12} = 3$ 
↔
 $\frac{5x}{12} \cdot \frac{12}{5} = \frac{12}{5} \cdot 3$ 
x =  $\frac{36}{5}$

## EXAMPLE 3 Try It! Use the Power of Power Property to Solve Equations With Rational Exponents

3. What is the solution of each equation?

a.  $256^{x+2} = 4^{3x+9}$

$4^{4(x+2)} = 4^{3x+9}$ 
multipl' expts
  
 $4^{4x+8} = 4^{3x+9}$ 
  
 $4x+8 = 3x+9$ 
Solve for x
  
 $-3x+8 = -3x+9$ 
  
 $x = 1$

b.  $(\frac{1}{8})^{\frac{x}{2}-1} = (\frac{1}{4})^{\frac{x}{3}}$

$(\frac{1}{2^3})^{\frac{x}{2}-1} = (\frac{1}{2^2})^{\frac{x}{3}}$ 
  
 $[\frac{1}{2}]^{3(\frac{x}{2}-1)} = [\frac{1}{2}]^{\frac{2x}{3}}$ 
same base
  
 $3(\frac{x}{2}-1) = \frac{2x}{3}$ 
mult by LCM: 6
  
 $\frac{3x}{2} - 3 = \frac{2x}{3}$ 
  
 $\frac{3x \cdot 6}{2} - 3 \cdot 6 = \frac{2x \cdot 6}{3}$ 
  
 $9x - 18 = 4x$ 
  
 $-4x + 18 = -4x + 18$ 
  
 $5x = 18$ 
  
 $x = \frac{18}{5}$

### HABITS OF MIND

Communicate Precisely When is the value of an expression undefined? Explain. MP.6

**EXAMPLE 4**  **Try It!** Use the Power of a Product Property to Solve Equations With Rational Exponents

4. When the side length of Blanket A is multiplied by  $2^{\frac{1}{2}}$  the result is 6 yards. Find the area of Blanket A.

**EXAMPLE 5**  **Try It!** Use the Quotient of Powers Property to Solve Equations With Rational Exponents

5. What is the value of  $x$  if the side length of Terrarium A is 3 times greater than the side length of Terrarium B?

**HABITS OF MIND**

**Look for Relationships** Can you use the same properties of exponents for expressions with rational exponents as you do when computing with integers? Explain.  MP.7

**Do You UNDERSTAND?**

- ESSENTIAL QUESTION** What are the properties of rational exponents and how are they used to solve problems?
- Communicate Precisely** A square has an area of 15 ft<sup>2</sup>. What are two ways of expressing its side lengths? © MP.6
- Look for Relationships** If  $3^x = 3^y$ , what is the relationship between  $x$  and  $y$ ? © MP.7
- Error Analysis** Corey wrote  $\sqrt[3]{4^2}$  as  $4^{\frac{2}{3}}$ . What error did Corey make? © MP.3
- Reason** When is it useful to have rational exponents instead of radicals? © MP.2
- Vocabulary** How are rational exponents different than whole number exponents? How are they the same?

**Do You KNOW HOW?**

Write each radical using rational exponents.  $a^{\frac{m}{n}} = \sqrt[n]{a^m}$   
 fraction power      radical

- $2\sqrt{7^1} \rightarrow 7^{\frac{1}{2}}$
- $\sqrt{15} \rightarrow 15^{\frac{1}{2}}$
- $\sqrt[3]{6^4} \rightarrow 6^{\frac{4}{3}}$
- $\sqrt[3]{2^3} \rightarrow 2^{\frac{3}{3}} \dots 2^1 \dots (2)$
- $\sqrt[4]{2^4} \rightarrow 2^{\frac{4}{4}} \dots 2^1 \dots 2$
- $\sqrt[2]{8^3} \rightarrow \sqrt[2]{\cancel{8} \cdot \cancel{8} \cdot 8} \rightarrow 8 \cdot 2\sqrt{2} \rightarrow 16\sqrt{2}$   
 leftover

Solve each equation.

- $(2^7)(2^{\frac{1}{2}}) = 2^5$
- $(4^2)(4^{\frac{3}{2}}) = 4^8$
- $64^{x+1} = 4^{x+7}$   
 $(4^3)^{x+1} = 4^{x+7} \rightarrow 3(x+1) = x+7 \rightarrow 3x+3 = x+7 \rightarrow 2x = 4 \rightarrow x = 2$
- $16^{(x-3)} = 2^{(x-6)}$   
 $(4^2)^{x-3} = 2^{x-6} \rightarrow 4(x-3) = x-6 \rightarrow 4x-12 = x-6 \rightarrow 3x = 6 \rightarrow x = 2$
- $(\frac{1}{27})^x = (\frac{1}{9})^{x+1}$
- $(\frac{1}{3})^{(x-4)} = (\frac{1}{27})^{x+1}$