Activity

6-3 Exponential Growth and Decay

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EXPLORE & REASON

Cindy is buying a new car and wants to learn how the value of her car will change over time. Insurance actuaries predict the future value of cars using depreciation functions. One such function is applied to the car whose declining value is shown.

Years After Purchase	Value
0 yr	$10,000
1 yr	$8,520
2 yr	$7,213
3 yr	$6,100
4 yr	$5,210

$-1480 $-1307 $-1113 $-890

A. Describe how the value of the car decreases from year to year.

non-linear depreciation ...

B. **Model With Mathematics** What kind of function would explain this type of pattern? MP.4

exponential decay

C. Given your answer to Part B, what is needed to find the function the actuary is using? Explain.

initial value &

percentage of depreciation ...

HABITS OF MIND

Make Sense and Persevere What is the constant ratio for the declining values? MP.1

100% ... $\underbrace{0 < x < 100\%}_{\text{between}}$

exponential growth → >100%

$f(x) = a \cdot b^x$

y, initial value, base

Notes

$f(x) = a \cdot (1+r)^x$ ← time

100%, growth rate (decimal)

ex1) Pop'l of Hillville grows annual rate of 15%. What is pop'l after 5 years.

5000, yearly

$f(x) = a(1+r)^x$

$f(x) = 5000(1+.15)^5$

$= 10056.78594 \rightarrow \boxed{10,057}$

$$$

Compound Interest

$A = P\left(1+\frac{r}{n}\right)^{nt}$

Final Amount, Principal (Initial amount), 100%, annual interest rate, # times interest is calculated per year, time

EXAMPLE 1

Try It! Exponential Growth

1. The population of Valleytown is also 5,000, with an annual increase of 1,000. Can the expected population for Valleytown be modeled with an exponential growth function? Explain.

linear, not exponential growth

EXAMPLE 2

Try It! Exponential Models of Growth

$A = P(1+\frac{r}{n})^{nt}$

2. a. What will be the difference after 15 years if the interest is compounded semiannually rather than quarterly?

n = 2

$A = 1{,}000{,}000\left(1+\frac{.08}{2}\right)^{2 \cdot 15}$

$= \$3,243,397.51$

n = 4

$A = 1{,}000{,}000\left(1+\frac{.08}{4}\right)^{4 \cdot 15}$

$= \$3,281,030.79$

$1,000,000

8% annual interest → .08

b. What would be the difference in after 15 years if the interest is compounded monthly rather than quarterly?

n = 12

$A = 1{,}000{,}000\left(1+\frac{.08}{12}\right)^{12 \cdot 15}$

$= \$3,306,921.48$

HABITS OF MIND

Communicate Precisely Explain why the total value increases the more times a value is compounded. MP.6

EXAMPLE 3

Try It! Exponential Decay

Video: 8192 views 1st day

3. Suppose the number of views decreases by 20% per day. In how many days will the number of views per day be less than 1,000?

20%

$f(x) = a \cdot b^x$

$= a \cdot (1-r)^x$

100%, decay

$f(x) = a \cdot (1-r)^x$

$= 8192(1-0.20)^x$

less than 1000 $= 8192(0.80)^x$

x	f(x)
0	8192
1	6553.6
2	5242.88
3	4194.304
⋮	
10	879.6

Notes

Assess

EXAMPLE 4 **Try It!** **Exponential Models of Decay**

4. How would the $\boxed{\text{average rate of change}}$ over the same intervals be affected if the population increased at a rate of 8%?

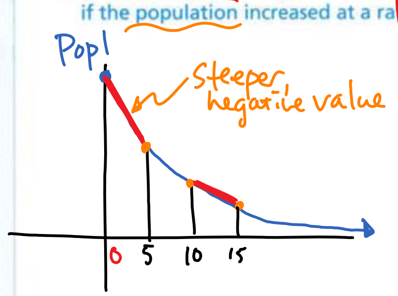
$a(1-r)^x$

$a(1-.68)^x$

$a(.92)^x$

Slope

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$



Steeper,
negative value

EXAMPLE 5 **Try It!** **Exponential Growth and Decay**

5. Explain how to use tables on a graphing calculator to answer this question.

GC

HABITS OF MIND

Model With Mathematics What are the key differences in the algebraic representations of exponential growth and decay? Explain. MP.4

Assess

Do You UNDERSTAND?

1. ESSENTIAL QUESTION What kinds of situations can be modeled with exponential growth or exponential decay?

2. **Vocabulary** What is the difference between simple interest and *compound interest*?

3. **Error Analysis** LaTanya says that the growth factor of $f(x) = 100(1.25)^x$ is 25%. What mistake did LaTanya make? Explain. MP.3

4. **Look for Relationships** Why is the growth factor $1 + r$ for an exponential growth function? MP.7

Do You KNOW HOW?

Write an exponential growth or decay function for each situation.

$y = a \cdot b^x$

$y = a(1 \pm r)^x$

initial

5. initial value of 100 increasing at a rate of 5%

r

0.05

$f(x) = \underbrace{100(1 + 0.05)^x}$

$f(x) = 100(1.05)^x$

6. initial value of 1,250 increasing at a rate of 25%

0.25

$f(x) = 1250(1 + 0.25)^x$

$f(x) = 1250(1.25)^x$

7. initial value of 512 decreasing at a rate of ~~58%~~ 47%

$f(x) = 512(1 - .47)^x$

$f(x) \quad 512(.53)^x$

8. initial value of 10,000 decreasing at a rate of 12%

$f(x) = 10000(1 - .12)^x$

$f(x) = 10000(.88)^x$

What if 10 years pass?

$x = 10$

$\ldots \quad 10000(.88)^{10} \approx 2785.01$

9. What is the difference in the value after 10 years of an initial investment of \$2,000 at 5% annual interest when the interest is compounded quarterly rather than ~~annually?~~ weekly

$A = P(1 + \frac{r}{n})^{nt}$

.05

n: 4

$A = 2000\left(1 + \frac{.05}{4}\right)^{4(10)}$

$= \$3287.24$

n: 52

$A = 2000\left(1 + \frac{.05}{52}\right)^{52(10)}$

$= \$3296.65$