6-4 Geometric Sequences

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EXPLORE & REASON

A seating plan is being designed for Section 12 of a new stadium.

2 seats

A. Describe the pattern.

Each row adds 3 seats starting at 2 (Row A)

B. Write an equation for this pattern.

$\boxed{3x-1}$

y-int →

x: 1, 2, 3, ...

x	y
0	-1
1	2
2	5
3	8
4	11
5	14

-3, +3, +3, +3, +3

C. Use Structure Row Z of Section 12 must have at least 75 seats. If the pattern continues, does this seating plan meet that requirement? Justify your answer. MP.7

26th letter: Z

$3(26)-1$

$\to 78-1 \to 77$

yes....

HABITS OF MIND

Use Appropriate Tools When is using a diagram the best tool to determine the number of seats in a given row? Explain. MP.1

Arithmetic Sequence vs. **Geometric Sequence**

Arithmetic Sequence:

- Common difference: $\boxed{d}$ (add/subt)
- explicit formula: $a_n = a_1 + (n-1)d$ or $a(n) = a_1 + (n-1)d$ (any n)
- recursive formula: $a_n = a_{n-1} + d$ (previous term)
- relies on previous term

Geometric Sequence:

- Common ratio: $\boxed{r}$ (mult/div)
- explicit formula: $a_n = a_1(r)^{n-1}$
- recursive formula: $a_n = r \cdot a_{n-1}$ (previous term)

recursive formula
• relies on ----> previous term

$a_n = a_{n-1} + d$

recursive formula $\{ a_n = r \cdot a_{n-1}$ previous term

Notes

d: r:

EXAMPLE 1 **Try It!** Identify Arithmetic and Geometric Sequences

1. Is each sequence an arithmetic or a geometric sequence? Explain.

a. 1, 2.2, 4.84, 10.648, 23.4256, ...

mult by 2.2? → Common ratio r: 2.2

→ geometric

b. 1, 75, 149, 223, 297, ...

+74 +74 +74 → Common difference d: 74

→ arithmetic

EXAMPLE 2 **Try It!** Write the Recursive Formula for a Sequence → relies on previous terms...

2. Write the recursive formulas for the geometric sequence 3,072, 768, 192, 48, 12, ...

·4 r: $\frac{1}{4}$

$a_n = r(a_{n-1})$

$\boxed{a_n = \frac{1}{4}(a_{n-1}),\ a_1 = 3072}$

common ratio previous

HABITS OF MIND

Make Sense and Persevere Explain why a common ratio in a geometric sequence cannot be zero. MP.1

...

EXAMPLE 3 **Try It!** Use the Explicit Formula

term #s
n: 1, 2, 3, 4, ...

3. What is the 12th term of the sequence described?
Initial condition is 3 ← a_1 (1st term)
recursive formula is $a_n = 6(a_{n-1})$ previous term

pattern ratio

explicit formula $a_n = a_1(r)^{n-1}$

12th term $\boxed{a_{12}} = 3(6)^{12-1}$

$= 1,088,391,168$

Notes

Assess

EXAMPLE 4 **Try It! Connect Geometric Sequences and Exponential Functions**

4. How many subscribers will there be in Week 9 if the initial number of subscribers is 10?

n:9

200%

a

• doubles every week...

mult by 2 → Common ratio

~~$f(x) = a \cdot b^x$~~

$$a_n = a_1 \cdot (r)^{n-1}$$

$$a_9 = 10 \cdot (2)^{9-1}$$

$$a_9 = 10 \cdot 2^8$$

$$= 10 \cdot 256$$

$$= \boxed{2560}$$

EXAMPLE 5 **Try It! Apply the Recursive and Explicit Formulas**

5. The formula $a_n = 1.5(a_{n-1})$ with an initial value of 40 describes a sequence. Use the explicit formula to determine the 5th term of the sequence.

previous term

recursive

150%

$$a_n = a_1 \cdot (r)^{n-1}$$

$$a_5 = 40(1.5)^{5-1}$$

$$= 40(1.5)^4$$

$$= 40(5.0625)$$

$$= \boxed{202.5}$$

HABITS OF MIND

Reason What is the relationship between the explicit formula and the recursive formula? Explain. MP.2

relies on term #
n: 1, 2, 3, ...

relies on previous term(s)

Assess

Do You UNDERSTAND?

1. ESSENTIAL QUESTION How are geometric sequences related to exponential functions?

2. **Vocabulary** How are *geometric sequences* similar to arithmetic sequences? How are they different?

3. **Error Analysis** For a geometric sequence with $a_1 = 3$ and a common ratio r of 1.25, Jamie writes $a_n = 1.25 \cdot (3)^{n-1}$. What mistake did Jamie make? MP.3

4. **Generalize** Is a sequence geometric if each term in the sequence is x times greater than the preceding term? MP.8

Do You KNOW HOW?

Determine whether the sequence is an arithmetic or a geometric sequence. If it is geometric, what is the common ratio?

d: current − previous
diff

r: $\frac{\text{current}}{\text{previous}}$
ratio

5. 30, 6, 1.2, 0.24, 0.048, …

÷5

$r: \frac{1}{5}$

geometric sequence

6. 0.5, 2, 8, 32, 148, …

×4

r: 4 geometric sequence

Write the recursive formula for each geometric sequence.

7. 640, 160, 40, 10, 2.5, …

÷4

$r: \frac{1}{4}$

$$\begin{cases} a_1 = 640 \\ a_n = \frac{1}{4}(a_{n-1}) \end{cases}$$

previous

8. 2, 5, 12.5, 31.25, 78.125, …

$r: \frac{5}{2}$

$$\begin{cases} a_1 = 2 \\ a_n = \frac{5}{2}(a_{n-1}) \end{cases}$$

9. What is the recursive formula for a sequence with the following explicit formula?
$a_n = 1.25 \cdot (3)^{n-1}$

a_1 r

n	a_n
1	1.25
⋮	⋮

$$\begin{cases} a_1 = 1.25 \\ a_n = 3(a_{n-1}) \end{cases}$$

10. A sequence has an initial value of 25 and a common ratio of 1.8. How can you write the sequence as a function? explicit

$$a_n = a_1(r)^{n-1}$$

$$\boxed{a_n = 25(1.8)^{n-1}}$$