

7-3

Multiplying Special Cases

EXPLORE & REASON

The table gives values for x and y and different expressions.

x	y	$(x-y)(x+y)$	x^2	y^2	(x^2-y^2)
7	4	$(7-4)(7+4)$	7^2	4^2	$(7^2-4^2) \rightarrow 49-16 \rightarrow 33$
6	2	$(6-2)(6+2)$	6^2	2^2	$(6^2-2^2) \rightarrow 36-4 \rightarrow 32$
3	9	$(3-9)(3+9)$	3^2	9^2	$(3^2-9^2) \rightarrow 9-81 \rightarrow -72$

A. Complete the table.



B. Describe any patterns you notice.

First
Outer
Inner
Last

$$(x-y)(x+y) = x^2 - y^2$$

$$x^2 + xy - xy - y^2$$

• Outer & Inner will cancel!

Product of a Sum & Difference

C. Use Structure Try substituting variable expressions of the form $7p$ and $4q$ for x and y . Does the pattern still hold? Explain. © MP7

$$(x-y)(x+y) \dots (7p-4q)(7p+4q)$$

F O I L

$$49p^2 - 16q^2$$

difference of Squares

- minus
- Square-rootable

ex)

$$9x^2 - 25,$$

$$144x^2y^2 - 81,$$

etc...

HABITS OF MIND

Generalize Did your exploration provide enough information establish a general rule? Explain. © MP8

EXAMPLE 1 Try It! Determine the Square of a Binomial

1. Find each product.

a. $(3x - 4)^2 \neq (3x)^2 - (4)^2$

expand and FOIL

$$(3x - 4)(3x - 4) \rightarrow 9x^2 - 24x + 16$$

$(3x)^2$ doubled $(-4)^2$

Square of a Binomial

Patterns?

$$(a+b)^2 = (a+b)(a+b)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = (a-b)(a-b)$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

b. 71^2

$$(70 + 1)^2 = 70^2 + 2(70)(1) + (1)^2$$

$$= 4900 + 140 + 1$$

$$= \boxed{5041}$$

EXAMPLE 2 Try It! Find the Product of a Sum and a Difference

2. Find each product.

a. $(2x - 4)(2x + 4) = (2x)^2 - (4)^2$

$$= a^2 - b^2$$

$$= \boxed{4x^2 - 16}$$

b. $56 \cdot 44$

$\underbrace{12 \text{ apart}}$

$$(50 + 6)(50 - 6) = 50^2 - 6^2$$

$$= a^2 - b^2$$

$$= 2500 - 36 = \boxed{2464}$$

Product of a
Sum & difference

$$(a+b)(a-b)$$

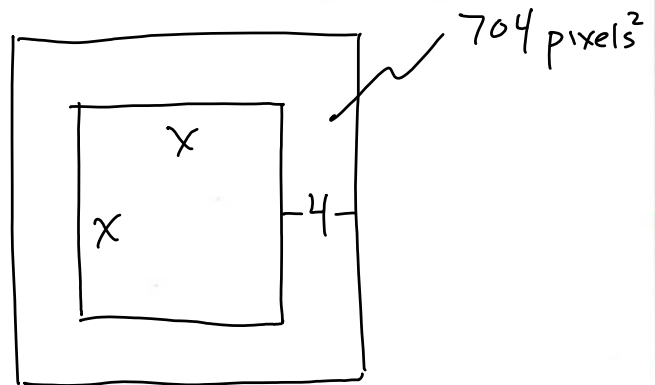
$$= a^2 - b^2$$

HABITS OF MIND

Use Appropriate Tools How do area models and algebraic expressions help you understand the patterns for the square of a binomial and for the product of a sum and a difference? © MMS

EXAMPLE 3 Try It! Apply the Square of a Binomial

3. What is the area of the square image if the area of the border is 704 square pixels and it is 4 pixels wide?



$$(x+8)(x+8) - x^2 = 704$$

$$\cancel{x^2} + 8x + 8x + 64 - \cancel{x^2} = 704$$

$$16x + 64 = 704$$

$$\begin{array}{r} 16x + 64 = 704 \\ \underline{-64} \quad \underline{-64} \\ 16x = 640 \\ \underline{16} \quad \underline{16} \\ x = 40 \end{array}$$

$$x = 40 \text{ pixels}$$

$$\dots 40(40) = \boxed{1600 \text{ pixels}^2}$$

HABITS OF MIND

Communicate Precisely What mathematical terms apply in this situation? MP.6

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** What patterns are there in the product of the square of a binomial and the product of a sum and a difference?

2. **Error Analysis** Kennedy multiplies $(x - 3)(x + 3)$ and gets an answer of $x^2 - 6x - 9$. Describe and correct Kennedy's error. **MP.3**

3. **Vocabulary** The product $(x + 6)(x - 6)$ is equivalent to an expression that is called the *difference of two squares*. Explain why the term *difference of two squares* is appropriate.

4. **Use Structure** Explain why the product of two binomials in the form $(a + b)(a - b)$ is a binomial instead of a trinomial. **MP.7**

Do You KNOW HOW?

Write each product in standard form.

5. $(x - 7)^2$ $(x-7)(x-7) = x^2 - 2(x)(7) + (7)^2$
 FOIL or use patterns $= x^2 - 14x + 49$

6. $(2x + 5)^2$ $(2x+5)(2x+5) = (2x)^2 - 2(2x)(5) + (5)^2$
 FOIL or use patterns $= 4x^2 - 20x + 25$

7. $(x + 4)(x - 4)$ $\rightarrow x^2 - (4)^2$
 FOIL or use patterns $\rightarrow x^2 - 16$

8. $(3y - 5)(3y + 5)$ $\rightarrow (3y)^2 - (5)^2$
 FOIL or use patterns $= 9y^2 - 25$

Use either the square of a binomial or the difference of two squares to find the area of each rectangle.

