

7-5

Factoring $x^2 + bx + c$

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EXPLORE & REASON

Consider the following puzzles.

- $1, 15$ $-1, -15$
- $3, 5$ $-3, -5$
- $-1, 15$ $-3, 5$
- $1, -15$ $3, -5$

A. Find the solutions to the four puzzles shown.

① $3 \cdot 5 = 15$ ② $-1 \cdot -15 = 15$ ③ $-5 \cdot 3 = -15$ ④ $-15 \cdot 1 = -15$
 $3 + 5 = 8$ $-1 + -15 = -16$ $-5 + 3 = -2$ $-15 + 1 = -14$

B. **Look for Relationships** Write a set of four number puzzles of your own that have the same structure as these four. Describe the pattern. © MP.7

⑤

HABITS OF MIND

Make Sense and Persevere Can you choose any pair of integers to create a solvable puzzle? Explain. © MP.1

$$\underline{ax^2} + \underline{bx} + \underline{c}$$

F	x^2
	36

x^2	
	28

	$x - 5$
x	$x^2 - 5x$
-3	$-3x + 15$

$$x^2 - 8x + 15$$

EXAMPLE 1 Try It! Understand Factoring a Trinomial

1. Write the factored form of each trinomial.

a. $x^2 + 13x + 36$

FOIL

$$(x+4)(x+9)$$

$$\begin{array}{r} ac \\ 36 \\ 1, 36 \\ 2, 18 \\ 3, 12 \\ 4, 9 \\ 6, 6 \end{array}$$

b. $x^2 + 11x + 28$

$$(x+4)(x+7)$$

$$\begin{array}{r} ac \\ 28 \\ 1, 28 \\ 2, 14 \\ 4, 7 \end{array}$$

EXAMPLE 2 Try It! Factor $x^2 + bx + c$, When $b < 0$ and $c > 0$

2. Write the factored form of each trinomial.

a. $x^2 - 8x + 15$

$$(x-3)(x-5)$$

$$\begin{array}{r} ac: 15 \\ -1, -15 \\ -3, -5 \end{array}$$

b. $x^2 - 13x + 42$

$$(x-6)(x-7)$$

$$\begin{array}{r} ac: 42 \\ -1, -42 \\ -2, -21 \\ -3, -14 \\ -6, -7 \end{array}$$

EXAMPLE 3 Try It! Factor $x^2 + bx + c$, When $c < 0$

3. Write the factored form of each trinomial.

a. $x^2 - 5x - 14$

$$(x+2)(x-7)$$

$$\begin{array}{r} ac: -14 \\ 1, -14 \\ 2, -7 \\ 7, -2 \\ 14, -1 \end{array}$$

b. $x^2 - 6x - 16$

$$(x+8)(x-2)$$

$$\begin{array}{r} ac: -16 \\ 1, -16 \\ 2, -8 \\ 4, -4 \\ 8, -2 \\ 16, -1 \end{array}$$

HABITS OF MIND

Use Structure If both b and c are negative, will the factors both be negative? Explain. MP

EXAMPLE 4

Try It! Factor a Trinomial With Two Variables

4. Write the factored form of each trinomial.

a. $x^2 + 12xy + 32y^2$

$x^2 + 12xy + 32y^2$

$(x + 4y)(x + 8y)$

	x	$8y$
x	x^2	$+8xy$
$4y$	$+4xy$	$+32y^2$

$12xy$

: ignore the other var temporarily...

b. $x^2 - 10xy + 21y^2$

$(x - 7y)(x - 3y)$

EXAMPLE 5

Try It! Apply Factoring Trinomials

5. What would be the dimensions of the larger wall area you would need if you used 11 of the 1 ft-by-1 ft units while keeping the other units the same?

HABITS OF MIND

Model With Mathematics How might factoring a trinomial into a pair of binomial factors relate to a situation in a physical world? © MP.4

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How does recognizing patterns in the signs of the terms help you factor polynomials?

2. **Error Analysis** A student says that since $x^2 - 5x - 6$ has two negative terms, both factors of c will be negative. Explain the error the student made. © MP.3

3. **Reason** What is the first step to factoring any trinomial? Explain. © MP.2

4. **Communicate Precisely** To factor a trinomial $x^2 + bx + c$, why do you find the factors of c and not b ? Explain. © MP.6

Do You KNOW HOW?

List the factor pairs of c for each trinomial.

5. $x^2 + 17x + 16$ 1, 16

6. $x^2 + 4x - 21$ -3, 7 1, 21
3, 7

For each trinomial, tell whether the factor pairs of c will be both positive, both negative, or opposite signs.

7. $x^2 - 11x + 10$ both negative

8. $x^2 + 9x - 10$ opposite signs

9. Complete the table for factoring the trinomial $x^2 - 7x + 12$.

Factors of 12	Sum of Factors
-1 and -12	? -13
-4 ? -3	-7
-2 and -6	-8