

$a \neq 1$

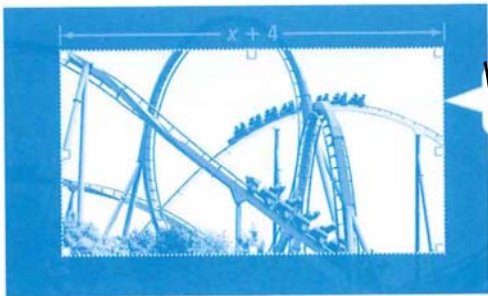
7-6

Factoring $ax^2 + bx + c$

PearsonRealize.com

EXPLORE & REASON

A website design company resizes rectangular photos so they fit on the screens of different devices.



Area: $(x+4)(x+12)$

Let's make a problem up...
 $(2x-5)(x+8)$

7.2

A. What expression represents the width of the photo?

$2x$	$2x^2$	$+16x$
-5	$-5x$	-40

$11x \Rightarrow 2x^2 + 11x - 40$

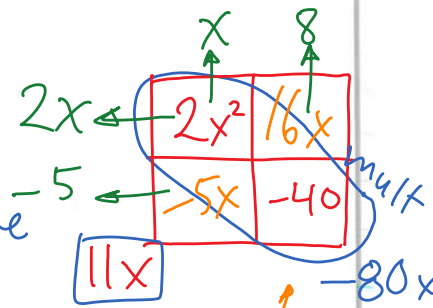
$ax^2 + bx + c$

B. Write three possible lengths and corresponding widths of the photo by substituting different values for x .

ac box method

Factor $2x^2 + 11x - 40$.

- ① 1st term \rightarrow 1st box
- Last term \rightarrow Last box
- Middle term \rightarrow Lower-Left outside corner



- ② Mult 1st & Last terms ($a \cdot c$)
- ③ Find factors of $a \cdot c$ whose sum is the middle term b
- ④ Place those factors in the box
- ⑤ GCF each row & column...

- $-80x^2$
- $-1, 80$
- $-2, 40$
- $-4, 20$
- $-5, 16$

HABITS OF MIND

Make Sense and Persevere Can you factor all trinomials of the form $ax^2 + bx + c$ or $(px + q)(sx + t)$, when $a, b, c, p, q, s,$ and t are integers? Explain. MP.1

* if leading term is negative, then the GCF will be negative.

EXAMPLE 1 Try It! Factor Out a Common Factor

1. Factor each trinomial.

a. $5x^2 - 35x + 50$

$$5(x^2 - 7x + 10)$$

$$5(x-5)(x-2)$$

b. $6x^3 + 30x^2 + 24x$

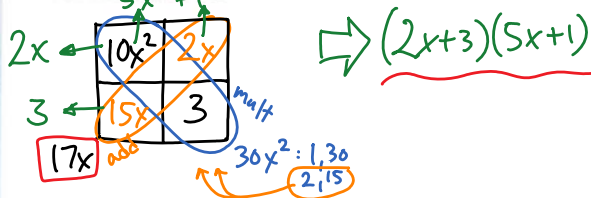
$$6x(x^2 + 5x + 4)$$

$$6x(x+4)(x+1)$$

EXAMPLE 2 Try It! Understand Factoring by Grouping

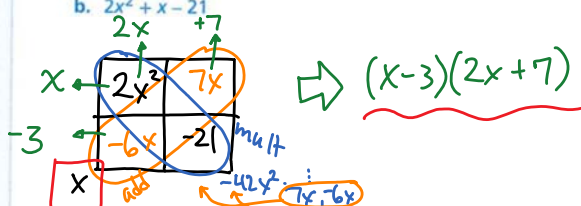
2. Factor each trinomial.

a. $10x^2 + 17x + 3$



$$\Rightarrow (2x+3)(5x+1)$$

b. $2x^2 + x - 21$



$$\Rightarrow (x-3)(2x+7)$$

HABITS OF MIND

Use Appropriate Tools Why is it helpful to factor out a GCF from a trinomial before factoring it as the product of binomials? Is it essential? Explain. © MP.5

EXAMPLE 3  **Try It!** Factor a Trinomial ~~Using Substitution~~3. Factor each trinomial ~~using substitution.~~

a. $2x^2 - x - 6$

	x	-2	
$2x \leftarrow$	$2x^2$	$-4x$	
$3 \leftarrow$	$3x$	-6	
$-x$			$= 12x^2 : -4x \ \& \ 3x$

$$(2x+3)(x-2)$$

b. $10x^2 + 3x - 1$

	$5x$	-1	
$2x \leftarrow$	$10x^2$	$-2x$	
$+1 \leftarrow$	$5x$	-1	
$3x$			$= 10x^2 : -2x \ \& \ 5x$

$$(2x+1)(5x-1)$$

HABITS OF MIND

Use Structure How does using substitution help make the process of factoring simpler?  **MP.7**

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** How is factoring a quadratic trinomial when $a \neq 1$ similar to factoring a quadratic trinomial when $a = 1$?

2. **Error Analysis** A student says that for $ax^2 + bx + c$ to be factorable, b must equal $a + c$. Explain the error in the student's thinking. © MP.3

3. **Reason** Suppose you can factor $ax^2 + bx + c$ as $(px + q)(sx + t)$, where $p, q, s,$ and t are integers. If $c = 1$, what do you know about the two binomial factors? © MP.2

4. **Reason** When factoring $ax^2 + bx + c$ by substitution, why is it acceptable to multiply the polynomial by a to start? © MP.2

5. **Construct Arguments** Felipe is factoring the expression $2x^2 - x - 28$. He knows $-x$ should be rewritten as $7x$ plus $-8x$, but he is not sure which order to place the terms in the expression. Explain to Felipe why it does not matter what order the terms are in. © MP.3

$ax^2 + bx + c$
Do You KNOW HOW?

List the factor pairs of ac for each trinomial.

6. $2x^2 + 7x + 4$ $a \cdot c = 2 \cdot 4 = 8$ $\begin{cases} 1 \cdot 8 \\ 2 \cdot 4 \end{cases}$

(Handwritten: 'a' points to 2, 'c' points to 4)

7. $12x^2 - 5x - 2$ $a \cdot c = 12(-2) = -24$ $\begin{cases} 1 \cdot -24 \\ 2 \cdot -12 \\ 3 \cdot -8 \\ 4 \cdot -6 \\ 6 \cdot -4 \\ 8 \cdot -3 \\ 12 \cdot -2 \\ 24 \cdot -1 \end{cases}$

(Handwritten: 'a' points to 12, 'c' points to -2; arrow points to 3 · -8)

Tell whether the terms of each trinomial share a common factor other than 1. If there is a common factor, identify it.

8. $15x^2 - 10x - 5$

GCF $\textcircled{5}$

9. $3x^3 - 2x^2 - 1$

$\textcircled{\text{none}}$

Rewrite the x -term in each trinomial to factor by grouping.

10. $35x^2 + 17x + 2$ $\begin{matrix} 5x & +1 \\ 7x & \leftarrow 35x^2 & 7x \\ 2 & \leftarrow 10x & 2 \end{matrix}$ $\textcircled{(7x+2)(5x+1)}$

(Handwritten: $70x^2 = 7x \cdot 10x$)

11. $12x^2 + 20x + 3$ $\begin{matrix} 6x & +1 \\ 2x & \leftarrow 12x^2 & 2x \\ 3 & \leftarrow 18x & 3 \end{matrix}$ $\textcircled{(2x+3)(6x+1)}$

(Handwritten: $36x^2 = 1 \cdot 36$, $2 \cdot 18$)

Factor each trinomial to find possible dimensions of each rectangle.

12. $A = 5x^2 + 17x + 6$ $\begin{matrix} 5x & +2 \\ x & \leftarrow 5x^2 & 2x \\ 3 & \leftarrow 15x & 6 \end{matrix}$ $\textcircled{(5x+2)(x+3)}$

(Handwritten: $30x^2 = 2x \cdot 15x$)

13. $A = 6x^2 + 7x - 5$ $\begin{matrix} 2x & -1 \\ 3x & \leftarrow 6x^2 & -3x \\ 5 & \leftarrow 10x & -5 \end{matrix}$ $\textcircled{(3x+5)(2x-1)}$

(Handwritten: $-30x^2 = -1 \cdot 30$, $-2 \cdot 15$, $-3 \cdot 10$)