

Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225,

$$ax^2 + bx + c = y$$

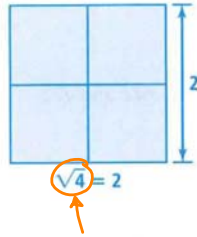
**9-3**  
Rewriting Radical Expressions  
PearsonRealize.com

**EXPLORE & REASON**

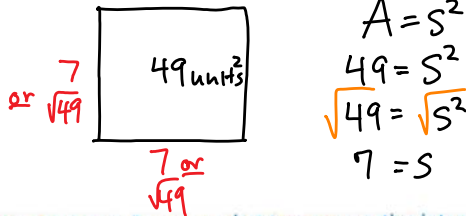
The table shows the relationship between the area of a square, the side length of the square, and the square root of the area. A square with an area of 4 and a side length of 2 is shown at the right.

radical  
Square Symbol

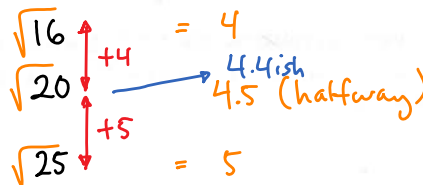
Area of Square (square units)	Area = $s^2$	Side Length, $s$ (units)
1	$1 = \sqrt{s}$	1
4	$4 = \sqrt{s}$	2
9	$9 = \sqrt{s}$	3
16	$16 = \sqrt{s}$	4
25	$25 = \sqrt{s}$	5



A. What is the side length of a square with an area of 49 square units?



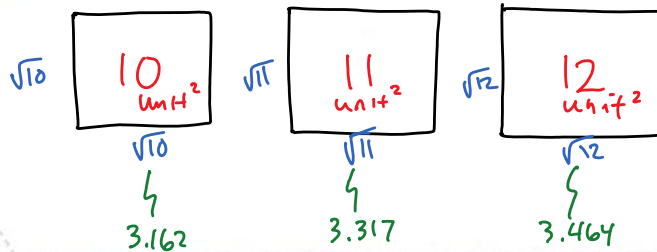
B. Use Structure Between what two consecutive integers is  $\sqrt{20}$ ? How do you know? © MP.7



Calc: 4.472 ...

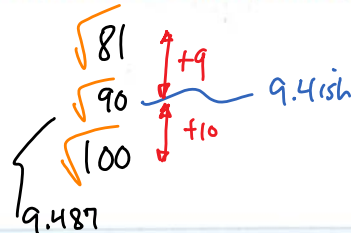
C. Think of three squares that have a side length between 3 and 4. What is the area of each square?

$$3 < s < 4$$



**HABITS OF MIND**

Communicate Precisely Is the  $\sqrt{90}$  closer to 9 or 10? Explain how you know. © MP.6



**EXAMPLE 1** Try It! Use Properties to Rewrite Radical Expressions

1. Compare each pair of radical expressions.

a.  $\sqrt{36}$  and  $\sqrt[3]{6}$

$\sqrt{6 \cdot 6}$  vs.  $\sqrt[3]{3 \cdot 3 \cdot 6}$  ←  $\sqrt{36}$  vs  $\sqrt[3]{54}$   
*not the same..* (with a sad face icon)

True or False

b.  $6\sqrt{2}$  and  $\sqrt{72}$

$\sqrt{6 \cdot 6 \cdot 2}$  vs.  $\sqrt{72}$   
*Same...*  
 Unsimplified radical

Simplified radical

3 is  $\sqrt{9}$   
 $\sqrt{3 \cdot 3}$

**EXAMPLE 2** Try It! Write Equivalent Radical Expressions

2. Rewrite each expression to remove perfect square factors other than 1 in the radicand.

a.  $\sqrt{44}$  =  $\sqrt{2 \cdot 2 \cdot 11}$

$2\sqrt{11}$  (Simplified radical)

b.  $3\sqrt{27}$  =  $3\sqrt{3 \cdot 3 \cdot 3}$   
 $= 3 \cdot 3\sqrt{3}$   
 $= 9\sqrt{3}$

Simplify  $\sqrt{88} = \sqrt{2 \cdot 2 \cdot 2 \cdot 11}$   
 $= 2\sqrt{2 \cdot 11}$   
 $= 2\sqrt{22}$

**EXAMPLE 3** Try It! Write Equivalent Radical Expressions With Variables

3. Rewrite each expression to remove perfect square factors other than 1 in the radicand.

a.  $\sqrt{25x^3}$  =  $\sqrt{5 \cdot 5 \cdot x \cdot x \cdot x}$   
 $= 5 \cdot x\sqrt{x}$

b.  $5\sqrt{4x^{17}}$  =  $5\sqrt{2 \cdot 2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$   
 $= 5 \cdot 2\sqrt{x^8 \cdot x}$   
 $= 10x^8\sqrt{x}$   
 17 ÷ 2 → 8½

→ pairs

**HABITS OF MIND**

**Generalize** How does the exponent of a variable help you to determine if the term is a perfect square? **MP8**

even #

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$$

Notes

Assess

**EXAMPLE 4** Try It! **Multiply Radical Expressions**

pairs...?

4. Write an expression for each product without perfect square factors in the radicand.

a.  $\frac{1}{2}\sqrt{21x^3} \cdot 4\sqrt{7x^2}$

$$\frac{1}{2} \cdot 4 \cdot \sqrt{21x^3} \cdot \sqrt{7x^2}$$

$$\frac{1}{2} \cdot 4 \cdot \sqrt{21x^3 \cdot 7x^2}$$

$$\frac{1}{2} \cdot 4 \cdot x \cdot x \cdot 7 \sqrt{3x}$$

$$14x^2\sqrt{3x}$$

b.  $2\sqrt{12x^9} \cdot \sqrt{18x^5}$

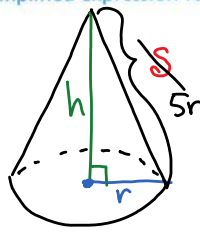
$$2\sqrt{12x^9 \cdot 18x^5} \rightarrow 2\sqrt{2 \cdot 6 \cdot 3 \cdot 6 \cdot x^{9+5}}$$

$$\rightarrow 2 \cdot 6 \cdot x^7 \sqrt{2 \cdot 3}$$

$$\rightarrow 12x^7\sqrt{6}$$

**EXAMPLE 5** Try It! **Write a Radical Expression**

5. Another cone has a slant height  $s$  that is 5 times the radius. What is the simplified expression for the height  $h$  in terms of the radius?



$$h^2 + r^2 = s^2$$

$$h^2 + r^2 = (5r)^2$$

$$h^2 + r^2 = 25r^2$$

$$h^2 = 24r^2$$

$$h = \sqrt{24r^2}$$

$$h = 2r\sqrt{6}$$

**HABITS OF MIND**

**Reason** When is a radical expression in simplest form? Justify your answer. **MP.2**

• no more factor pairs in the radical ...

**Do You UNDERSTAND?**

- ESSENTIAL QUESTION** How does rewriting radicals in different forms help you communicate your answer?
- Vocabulary** State the *Product Property of Square Roots* in your own words.
- Communicate Precisely** Write an expression for  $\sqrt{32}$  without any perfect square factors in the radicand. Explain your steps. © MP.6
- Error Analysis** Rikki says that the product  $\sqrt{3x^3} \cdot \sqrt{x}$  is  $3x^2$ . Explain Rikki's error and write the correct product. © MP.3
- Construct Arguments** Is  $\sqrt{45}$  in simplest form? Explain. © MP.3
- Make Sense and Persevere** Describe how you would simplify an expression so that there are no perfect square factors in the radicand. © MP.1

**Do You KNOW HOW?**

Factor each radicand using the **Product Property of Square Roots**

- $\sqrt{80}$ 

$$\begin{matrix} 8 < \begin{matrix} 2 \\ 2 \end{matrix} \\ 10 < \begin{matrix} 2 \\ 2 \end{matrix} \end{matrix} \quad \sqrt{\cancel{2}\cancel{2}\cancel{2}5} \rightarrow 2 \cdot 2\sqrt{5}$$

$$\rightarrow 4\sqrt{5}$$
- $\sqrt{x^7}$ 

$$\frac{7}{2} \leftarrow 3\frac{1}{2} \rightarrow x^3\sqrt{x^1}$$
- $\sqrt{40x^4}$ 

$$\begin{matrix} 4 < \begin{matrix} 2 \\ 2 \end{matrix} \\ 10 < \begin{matrix} 2 \\ 2 \end{matrix} \end{matrix} \quad \sqrt{\cancel{2}\cancel{2}5\cancel{x}\cancel{x}x^2} \rightarrow 2xx\sqrt{2 \cdot 5}$$

$$\rightarrow 2x^2\sqrt{10}$$
- $\sqrt{11x^5}$ 

$$\frac{5}{2} \leftarrow 2\frac{1}{2} \rightarrow x^2\sqrt{11x^1}$$
- $\sqrt{200}$ 

$$\begin{matrix} 2 \\ 100 \end{matrix} \quad \sqrt{2 \cdot 100} \rightarrow 10\sqrt{2}$$
- $8\sqrt{8}$ 

$$\begin{matrix} 2 \\ 2 \\ 2 \end{matrix} \quad 8\sqrt{\cancel{2}\cancel{2}2} \rightarrow 8 \cdot 2\sqrt{2} \rightarrow 16\sqrt{2}$$

Write an expression for each product without a perfect square factor other than 1 in the radicand.

- $4\sqrt{3x^3} \cdot 3\sqrt{2x^2}$ 

$$4 \cdot 3 \sqrt{\cancel{3}x^3 \cdot \cancel{2}x^2} \rightarrow 12\sqrt{6x^5}$$

$$\rightarrow 12x^2\sqrt{6x^1}$$
- $x\sqrt{2x^5} \cdot 2x\sqrt{8x}$ 

$$2xx\sqrt{\cancel{2}x^5 \cdot 8x} \rightarrow 2xx \cdot 2 \cdot 2\sqrt{x^{5+1}}$$

$$\rightarrow 8x^2\sqrt{x^6}$$

$$\rightarrow 8x^2x^3 = 8x^5$$
- $\sqrt{7x} \cdot 3\sqrt{10x^7}$ 

$$3\sqrt{7x \cdot 10x^7} \rightarrow 3x^4\sqrt{70}$$

Compare each pair of radical expressions by writing each expression as a product of square roots in simplest form.

- $\sqrt{72}$  and  $2\sqrt{50}$ 

$$\sqrt{50 \cdot 2 \cdot 2} \rightarrow \sqrt{200}$$

$$\sqrt{72} \neq 2\sqrt{50}$$
- $\sqrt{\frac{5}{28}}$  and  $\sqrt{119}$ 

$$\sqrt{5 \cdot 5 \cdot 28} \rightarrow \sqrt{700}$$

$$5\sqrt{28} \neq \sqrt{119}$$

Write each expression so there are no perfect square factors other than 1 in the radicand.

- $10x^8$ 

$$10x^4$$
- $4x^2y\sqrt{3x^4y^6}$ 

$$4x^2y \cdot x^2y^3\sqrt{2} \rightarrow 4x^4y^4\sqrt{2}$$