

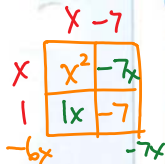
9-5

Completing the Square

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CRITIQUE AND EXPLAIN

Enrique and Nadeem used different methods to solve the equation $x^2 - 6x + 9 = 16$.



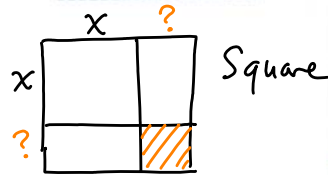
Enrique
 $x^2 - 6x + 9 = 16$
 $x^2 - 6x - 7 = 0$ • factored
 $(x-7)(x+1) = 0$
 $x-7=0$ OR $x+1=0$
 $x=7$ OR $x=-1$
 The solutions are 7 and -1

Nadeem
 $x^2 - 6x + 9 = 16$
 $(x-3)^2 = 16$
 $x-3 = \pm 4$ ← square root
 $x-3 = 4$ OR $x-3 = -4$
 $x = 7$ OR $x = -1$
 The solutions are 7 and -1.

A. Critique Enrique's work. If his method is valid, explain the reasoning he used. If his method is not valid, explain why not.



B. Critique Nadeem's work. If his method is valid, explain the reasoning he used. If his method is not valid, explain why not.



C. Use Structure Can you use either Enrique's or Nadeem's method to solve the equation $x^2 + 10x + 25 = 3$? Explain. © MP7

Handwritten work for problem C:

Enrique's method fails...

$$x^2 + 10x + 25 = 3$$

$$x^2 + (10x) + 22 = 0$$

Handwritten notes: $1, 22$ and $2, 11$ with arrows pointing to the constant term and the linear coefficient.

Nadeem's method:

$$x^2 + 10x + 25 = 3$$

Handwritten notes: $5+5$ and $5 \cdot 5$ with arrows pointing to the $10x$ and 25 terms.

$$(x+5)(x+5) = 3$$

$$(x+5)^2 = 3$$

$$\sqrt{(x+5)^2} = \pm\sqrt{3}$$

$$x+5 = \pm\sqrt{3}$$

Final solution:

$$x = -5 \pm \sqrt{3}$$

or $\pm\sqrt{3} - 5$

Handwritten note: decimals $-3.268, -6.732$

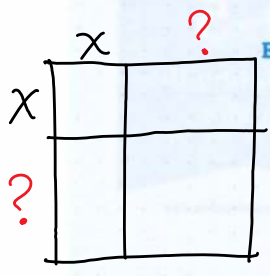
HABITS OF MIND

Reason Explain how the Properties of Equality are used to solve equations. © MP2

- apply the operation(s) to both sides....
- $+, -, \times, \div, \sqrt{\quad}, \sqrt{\quad}^2$

$$ax^2 + bx + c$$

* $a=1$ & * $\left(\frac{b}{2}\right)^2$



EXAMPLE 1 Try It! Complete the Square

1. What value of c completes the square?

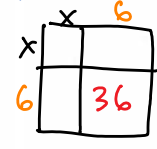
a. $x^2 + 12x + c$

$a=1$

$\left(\frac{12}{2}\right)^2 \rightarrow 6^2 \rightarrow 36$

half of b Square it... PST? Perfect Square trinomial

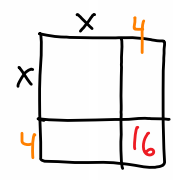
$$x^2 + 12x + 36$$



b. $x^2 + 8x + c$

$a=1$

$\left(\frac{8}{2}\right)^2 \rightarrow 4^2 \rightarrow 16$



Expressions

EXAMPLE 2 Try It! Solve $x^2 + bx + c = 0$

2. What are the solutions of each quadratic equation? Solve by completing the square.

a. $x^2 + 10x - 9 = 0$

$a=1$

$x^2 + 10x + 25 = 9 + 25$

$x^2 + 10x + 25 = 34$

balance the equation!

- move c
- PST

$(x+5)(x+5) = 34$

$(x+5)^2 = 34$

$\sqrt{(x+5)^2} = \pm\sqrt{34}$

$x+5 = \pm\sqrt{34}$

$x = -5 \pm \sqrt{34}$

Equation
• see-saw

b. $x^2 - 8x - 9 = 0$

$x^2 - 8x + 16 = 6 + 16$

$x^2 - 8x + 16 = 22$

$(x-4)(x-4) = 22$

$(x-4)^2 = 22$

$\sqrt{(x-4)^2} = \pm\sqrt{22}$

$x-4 = \pm\sqrt{22}$

$x = 4 \pm \sqrt{22}$

EXAMPLE 3 Try It! Complete the Square When $a \neq 1$ Initially

3. A maze and walkway with the same total area of 5,616 square yards has a walkway that is one yard wide. What are the dimensions of this maze?



HABITS OF MIND

Reason Is it possible for the value used to complete the square to be negative? Explain. © MP.2

Notes
 Complete the Square
 • $a=1$
 • $(\frac{b}{2})^2$

$y = ax^2 + bx + c$
 Standard Form

Complete the Square

$y = a(x-h)^2 + k$
 Vertex Form

EXAMPLE 4

Try It! Use Completing the Square to Write a Quadratic Function in Vertex Form (h, k)

4. What is the vertex form of each function?

a. $y = x^2 - 2x + 3$

$y = x^2 - 2x + 1 + 3 - 1$

$(\frac{-2}{2})^2$
 PST

$y = (x-1)(x-1) + 2$

$y = (x-1)^2 + 2$

b. $y = x^2 + 6x + 25$

$y = x^2 + 6x + 9 + 25 - 9$

$(\frac{6}{2})^2$
 PST

$y = (x+3)(x+3) + 16$

$y = (x+3)^2 + 16$

EXAMPLE 5

Try It! Write Vertex Form When $a \neq 1$

5. Find the minimum value of the function $y = 7x^2 + 168x + 105$.

Real World Problem

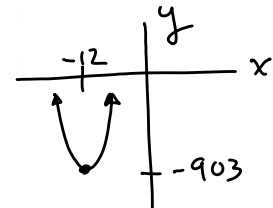
$y = 7(x^2 + 24x + 144) + 105 - 7(144)$

$y = 7(x+12)(x+12) + 105 - 1008$

$a(x-h)^2 + k$
 $y = 7(x+12)^2 - 903$

$V: (-12, -903)$

Minimum



HABITS OF MIND

Communicate Precisely How is completing the square related to writing the vertex form of a function? Explain. MP.6

$y = a(x-h)^2 + k$

Do You UNDERSTAND?

- ESSENTIAL QUESTION** How is the technique of completing the square helpful for analyzing quadratic functions?
- Vocabulary** Why does it make sense to describe adding 25 to $x^2 + 10x$ as *completing the square*?
- Error Analysis** A student began solving $x^2 + 8x = 5$ by writing $x^2 + 8x + 16 = 5$. Explain the error the student made. © MP.3
- Communicate Precisely** How is changing a quadratic function from standard form to vertex form like solving a quadratic equation by completing the square? How is it different? © MP.6
- Look for Relationships** Why is it necessary for the coefficient of x^2 to be 1 before completing the square? © MP.7

Do You KNOW HOW?

Find the value of c that makes each expression a perfect-square trinomial.

6. $x^2 + 26x + c$ 169

(Handwritten: $(\frac{26}{2})^2$)

7. $x^2 + 2x + c$ 1

(Handwritten: $(\frac{2}{2})^2$)

8. $x^2 + 18x + c$ 81

(Handwritten: $(\frac{18}{2})^2$)

Solve each equation.

9. $x^2 + 8x = -1$

(Handwritten: $x^2 + 8x + 16 = -1 + 16$)

(Handwritten: $(x+4)(x+4) = 15$)

(Handwritten: $(x+4)^2 = 15$)

(Handwritten: $\sqrt{(x+4)^2} = \pm\sqrt{15}$)

(Handwritten: $x+4 = \pm\sqrt{15}$)

(Handwritten: $x = -4 \pm\sqrt{15}$)

10. $(2x^2 - 24x - 4 = 0) \div 2$

(Handwritten: $x^2 - 12x + 36 = -2 + 36$)

(Handwritten: $(x-6)(x-6) = 38$)

(Handwritten: $(x-6)^2 = 38$)

(Handwritten: $\sqrt{(x-6)^2} = \pm\sqrt{38}$)

(Handwritten: $x-6 = \pm\sqrt{38}$)

(Handwritten: $x = 6 \pm\sqrt{38}$)

11. $x^2 - 4x = 7$

(Handwritten: $x^2 - 4x + 4 = 7 + 4$)

(Handwritten: $(x-2)(x-2) = 11$)

(Handwritten: $(x-2)^2 = 11$)

(Handwritten: $\sqrt{(x-2)^2} = \pm\sqrt{11}$)

(Handwritten: $x-2 = \pm\sqrt{11}$)

(Handwritten: $x = 2 \pm\sqrt{11}$)

Write each function in vertex form.

12. $y = x^2 + 4x - 5$

(Handwritten: $y = (x+2)(x+2) - 9$)

(Handwritten: $y = (x+2)^2 - 9$)

13. $y = 5x^2 - 10x + 7$

(Handwritten: $5(x^2 - 2x + 1) + 7 - 5(1)$)

(Handwritten: $5(x-1)(x-1) + 7 - 5$)

(Handwritten: $5(x-1)^2 + 2 = y$)

14. $y = x^2 + 8x - 15$

(Handwritten: $x^2 + 8x + 16 - 15 - 16$)

(Handwritten: $y = (x+4)(x+4) - 31$)

(Handwritten: $y = (x+4)^2 - 31$)