

9-6
 The Quadratic Formula and the Discriminant
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EXPLORE & REASON

Three quadratic equations are shown on the whiteboard.

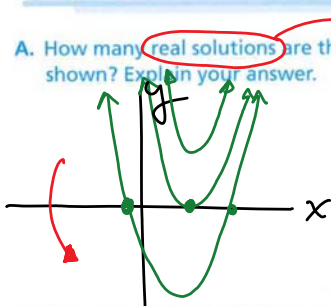
$$x^2 - 6x + 12 = 0$$

$$x^2 - 6x + 9 = 0$$

$$x^2 - 6x - 5 = 0$$

y-intercepts

A. How many real solutions are there for each of the quadratic equations shown? Explain your answer.



B. Use **Appropriate Tools** Use your graphing calculator to graph the related function for each equation. What are the function equations for each graph's reflection over the x-axis? Explain how you found the function equations.

MP.5

reflect vertically : $a < 0$ (negative)

C. What do you notice about the graphs that have zero x-intercepts?
 One x-intercept? Two x-intercepts?

Hmm.....

HABITS OF MIND

Reason How can the number of solutions to a quadratic equation be determined by inspecting its graph? MP.2

2, 1, or 0.....

$$y = ax^2 + bx + c$$

set $y=0$

Quadratic
Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXAMPLE 1 **Try It!** Derive the Quadratic Formula

1. What is the maximum number of solutions the quadratic formula can give? Explain

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

at most
two real
solutions

EXAMPLE 2 **Try It!** Use the Quadratic Formula

2. Find the solutions of each equation using the quadratic formula.

a. $21 - 4x = x^2 + 4x - 21$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(4)^2 - 4(1)(-21)}}{2(1)}$$

$$0 = x^2 + 4x - 21$$

$a=1$
 $b=4$
 $c=-21$

$$= \frac{-4 \pm \sqrt{16 + 84}}{2} = \frac{-4 \pm \sqrt{100}}{2} = \frac{-4 \pm 10}{2}$$

$\frac{-4+10}{2}$ or $\frac{-4-10}{2}$

3 or -7

b. $x^2 - 2x - 24 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-24)}}{2(1)} = \frac{2 \pm \sqrt{4 + 96}}{2} = \frac{2 \pm \sqrt{100}}{2}$$

$a=1$
 $b=-2$
 $c=-24$

$$= \frac{2 \pm 10}{2} = \frac{2+10}{2} \text{ or } \frac{2-10}{2}$$

6 or -4

HABITS OF MIND

Look for Relationships Using the quadratic formula, how can you tell when a quadratic equation has only one solution? MP.7

$$\dots \pm \sqrt{0}$$

double root

EXAMPLE 3 Try It! Find Approximate Solutions

3. The height of another frog over time is modeled by the function $y = -16t^2 + 10t + 0.3$. How many seconds is this frog in the air before landing on the ground? Round your answer to the nearest hundredth.

$$t = \frac{-(10) \pm \sqrt{(10)^2 - 4(-16)(0.3)}}{2(-16)}$$

$$t = \boxed{0.6536} \text{ or } -0.0287$$

EXAMPLE 4 Try It! Understand and Use the Discriminant

4. Use the discriminant to find the number of roots of each equation.

a. $x^2 - 10x + 25 = 0$

$a = 1$

$b = -10$

$c = 25$

$b^2 - 4ac$

$(-10)^2 - 4(1)(25)$

$100 - 100$

$\boxed{0} \rightarrow 1 \text{ real solution}$

b. $-x^2 - 6x - 10 = 0$

$a = -1$

$b = -6$

$c = -10$


$b^2 - 4ac$


$(-6)^2 - 4(-1)(-10)$


$36 - 40$

$\boxed{-4} \rightarrow 0 \text{ real solutions}$

$$-b \pm \sqrt{b^2 - 4ac}$$

$b^2 - 4ac > 0$: $2a$ ^{Pos.} 
2 real solutions

$b^2 - 4ac = 0$: ^{zero} 
1 real solution

$b^2 - 4ac < 0$: ^{neg.} 
0 real solutions

HABITS OF MIND

Reason If the equation $4x^2 - bx + 9 = 0$ has only 1 solution, what is the value of b ?  MP.2

$a = 4$

$\text{neg } b = ?$

$c = 9$

$$b^2 - 4ac = 0$$

$$b^2 - 4(4)(9) = 0$$

$$b^2 - 144 = 0$$

$$b^2 = 144$$

$$b = \pm 12$$

Do You UNDERSTAND?

1. **ESSENTIAL QUESTION** When should you use the quadratic formula to solve equations?

2. **Reason** What value of b^2 is needed for there to be exactly one real solution of a quadratic equation? Explain. © MP.2

3. **Vocabulary** How are the roots of a quadratic equation related to its *discriminant*?

4. **Error Analysis** A student says that the quadratic formula cannot be used to solve $-23x^2 + 5 = 0$. Explain the error the student made. © MP.3

5. **Reason** When is completing the square better than using the quadratic formula? © MP.2

Do You KNOW HOW?

Identify a , b , and c in each of the quadratic equations.

6. $4x^2 + 2x - 1 = 0$

$a = 4$

$b = 2$

$c = -1$

7. $-x^2 + 31x + 7 = 0$

$a = -1$

$b = 31$

$c = 7$

8. $2x^2 - 10x - 3 = 0$

$a = 2$

$b = -10$

$c = -3$

9. $x^2 + x - 1 = 0$

$a = 1$

$b = 1$

$c = -1$



Given the discriminant of a quadratic equation, determine the number of real solutions.

10. 8

2 reals

11. -3

0 reals

12. 0

1 real

13. 1

2 reals