

TOPIC
1

Topic Review

? TOPIC ESSENTIAL QUESTION

1. What general strategies can you use to solve simple equations and inequalities?

Vocabulary Review

Choose the correct term to complete each sentence.

- An equation rule for a relationship between two or more quantities is a(n) formula.
- A combination of two or more inequalities using the word and or the word or is a(n) compound inequality.
- Any of the distinct objects of a set is called a(n) element.
- If each element of B is also an element of A, B is a(n) subset of A.
- A well-defined collection of elements is a(n) set.
- An equation where letters are used for constants and variables is a(n) literal equation.
- An equation that is true for all values of the variable is a(n) identity.

- compound inequality ✓
- element ✓
- formula ✓
- identity ✓
- literal equation ✓
- set ✓
- subset ✓

Concepts & Skills Review

LESSON 1-1 Operations on Real Numbers

Quick Review

Sums, differences, and products of rational numbers are rational. Quotients of rational numbers (when they are defined) are rational.

The sum and difference of a rational number and an irrational number are irrational. The product and quotient (when defined) of a rational number and an irrational number are irrational, except when the rational number is 0.

Example

Let $a, b, c,$ and d be integers, with $b \neq 0$ and $d \neq 0$. Is the sum of $\frac{a}{b}$ and $\frac{c}{d}$ rational or irrational? Is their product rational or irrational?

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

The sum and product are both rational.

Practice & Problem Solving

9. Give an example of two irrational numbers whose product is rational.

For each number, determine whether it is an element of the real numbers, irrational numbers, rational numbers, integers, or whole numbers. List all that apply.

10. 13.9 11. $\sqrt{49}$ 12. -48

Order from least to greatest.

13. $0.3\bar{6}, \sqrt{15}, \sqrt{\frac{17}{3}}$ 14. $\frac{29}{12}, 2.4, \sqrt{5.65}$

15. **Make Sense and Persevere** Taylor uses tape to mark a square play area in the basement for her daughter. The area measures 28 ft². Is the side length of the square rational or irrational? Explain.

LESSON 1-2 Solving Linear Equations

Quick Review

You can use properties of equality to solve linear equations. Use the Distributive Property and combine like terms, when needed.

Example

Solve $\frac{2}{3}(6x - 15) + 5x = 26$.

$$\begin{aligned} \frac{2}{3}(6x - 15) + 5x &= 26 \\ 4x - 10 + 5x &= 26 && \text{Distributive Property} \\ 9x - 10 &= 26 && \text{Combine like terms.} \\ 9x - 10 + 10 &= 26 + 10 && \text{Add 10 to each side.} \\ 9x &= 36 && \text{Simplify.} \\ \frac{9x}{9} &= \frac{36}{9} && \text{Divide each side by 9.} \\ x &= 4 && \text{Simplify.} \end{aligned}$$

Practice & Problem Solving

16. **Use Structure** What property would you use first to solve $\frac{1}{2}x - 6 = 10$? Explain.

Solve each equation. +6 /16 Addition Prop of Equality

17. $3(2x - 1) = 21$
18. $100 = 8(4t - 5)$
19. $\frac{5}{8} = \frac{3}{4}b - \frac{7}{12}$
20. $1.045s + 0.068 = 15.743$

21. **Model With Mathematics** The price for an adult movie ticket is $1\frac{1}{3}$ more than a movie ticket for a child. Ines takes her daughter to the movie, buys a box of popcorn for \$5.50, and spends \$26.50. Write and solve an equation to find the prices for each of their movie tickets.

PE MD AS

Undo operations

LESSON 1-3 Solving Equations with a Variable on Both Sides

Quick Review

To solve equations with a variable on both sides, rewrite the equation so that all the variable terms are on one side of the equation and the constants are on the other. Then solve for the value of the variable.

Example

Solve $5x - 48 = -3x + 8$.

$$\begin{aligned} 5x - 48 &= -3x + 8 \\ 5x - 48 + 3x &= -3x + 8 + 3x && \text{Add 3x to each side.} \\ 8x - 48 &= 8 \end{aligned}$$

Practice & Problem Solving

22. **Error Analysis** Describe and correct any errors a student may have made when solving the equation $0.6(y - 0.2) = 3 - 0.2(y - 1)$.

$$\begin{aligned} 0.6(y - 0.2) &= 3 - 0.2(y - 1) \\ 0.6y - 0.12 &= 3.2 - 0.2y \\ 100(0.6y - 0.12) &= 10(3.2 - 0.2y) \\ 60y - 12 &= 32 - 2y \\ 60y - 12 + 2y &= 32 + 12 - 2y + 2y \\ 62y &= 42 \\ y &= \frac{21}{31} \end{aligned}$$

Handwritten student work for solving equations:

17) $3(2x - 1) = 21$
 $6x - 3 = 21$
 $6x = 24$
 $x = 4$

18) $100 = 8(4t - 5)$
 $100 = 32t - 40$
 $140 = 32t$
 $4.375 = t$

19) $\frac{5}{8} = \frac{3}{4}b - \frac{7}{12}$ (Multiply LCD 24)
 $24(\frac{5}{8}) = 24(\frac{3}{4}b - \frac{7}{12})$
 $15 = 18b - 14$
 $29 = 18b$
 $\frac{29}{18} = b$

23) $21 - 4x = 4x + 21$
 $21 = 8x + 21$
 $0 = 8x$
 $0 = x$

24) $6b - 27 = 3(5b - 2)$
 $6b - 27 = 15b - 6$
 $-27 = 9b - 6$
 $-21 = 9b$
 $-3 = b$

Example

Solve $5x - 48 = -3x + 8$.

$5x - 48 = -3x + 8$

$8x - 48 = 8$ Simplify.

$8x - 48 + 48 = 8 + 48$ Add 48 to each side.

$8x = 56$ Simplify.

$\frac{8x}{8} = \frac{56}{8}$ Divide each side by 8.

$x = 7$ Simplify.

PE
MD
AS

Solve

$60y - 12 = 32 - 2y$
 $60y - 12 + 12 + 2y = 32 + 12 - 2y + 2y$
 $62y = 42$
 $y = \frac{21}{31}$

Solve each equation.

- 23. $21 - 4x = 4x + 21$
- 24. $6b - 27 = 3(5b - 2)$
- 25. $0.45(t + 8) = 0.6(t - 3)$
- 26. **Construct Arguments** Aaron can join a gym that charges \$19.99 per month, plus an annual \$12.80 fee, or he can pay \$21.59 per month. He thinks the second option is better because he plans to use the gym for 10 months. Is Aaron correct? Explain.

$0 = \frac{8x}{8}$
 $0 = x$

or $-21 = \frac{9b}{9}$
 $-\frac{7}{3} = b$

LESSON 1-4 Literal Equations and Formulas

Quick Review

You can use properties of equality to solve literal equations for a specific variable. You can use the rewritten equation as a formula to solve problems.

Example

Find the height of a cylinder with a volume of 1,650 cm³ and a radius of 6 cm.

Rewrite the formula for the volume of a cylinder in terms of h.

$V = \pi r^2 h$

$\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$

$\frac{V}{\pi r^2} = h$

Find the height of the cylinder. Use 3.14 for pi.

$h = \frac{V}{\pi r^2}$

$h = \frac{1,650}{(3.14)(6)^2} = \frac{1,650}{(3.14)(36)} = \frac{1,650}{113.04} \approx 14.60$

The height of the cylinder is about 14.60 cm.

Practice & Problem Solving

27. **Error Analysis** Describe and correct the error a student made when solving $a = \frac{3}{4}(b + 5)$ for b.

$a = \frac{3}{4}(b + 5)$
 $\frac{4}{3}a = \frac{3}{4}(b + 5) \cdot \frac{4}{3}$ reciprocal
 $\frac{4}{3}a = b + 5$ error
 $b = \frac{4}{3}a - 5$

Solve each equation for the given variable.

- 28. $xy = k$; y
- 29. $a = \frac{2}{b} + 3c$; c
- 30. $6(2c + 3d) = 5(4c - 3d)$; d

31. **Model With Mathematics** The formula for average acceleration is $a = \frac{V_f - V_i}{t}$, where V_f is the final velocity, V_i is the initial velocity, and t is the time in seconds. Rewrite the equation as a formula for the final velocity, V_f . What is the final velocity when a person accelerates at 2 ft/s² for 5 seconds after an initial velocity of 4 ft/s?

LESSON 1-5 Solving Inequalities in One Variable

Quick Review

The same strategies used for solving multistep equations can be used to solve multistep inequalities. When multiplying or dividing by a negative value, reverse the inequality symbol.

Example

Solve $-2(6x + 5) \leq 74$. Graph the solution.

$-2(6x + 5) \leq 74$

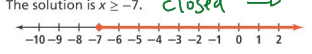
$-12x - 10 \leq 74$ Distributive Property

$-12x - 10 + 10 \leq 74 + 10$ Add 10 to each side.

$-12x \geq 84$ Divide each side by -12.

$x \geq -7$ Simplify.

The solution is $x \geq -7$. closed \rightarrow



Practice & Problem Solving

32. **Use Structure** Write an inequality that represents the graph.



Solve each inequality and graph the solution.

- 33. $x + 8 > 11$
- 34. $4x + 3 \leq -6$
- 35. $2.4x - 9 < 1.8x + 6$
- 36. $3x - 8 \geq 4(x - 1.5)$

37. **Make Sense and Persevere** Neil and Yuki run a data entry service. Neil starts at 9:00 a.m. and can type 45 words per minute. Yuki arrives at 10:30 a.m. and can type 60 words per minute. Write and solve an inequality to find at what time Yuki will have typed more words than Neil. Let x represent the time in minutes.

LESSON 1-6 Compound Inequalities

Quick Review

When a compound inequality uses the word and, the solution must make both inequalities true. If a compound inequality uses the word or, the solution must make at least one of the inequalities true.

Example

Solve $-24 < 4x - 4 < 4$. Graph the solution.

Separate the inequality and solve each separately.

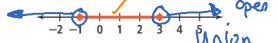
$-24 < 4x - 4$ and $4x - 4 < 4$

$-24 + 4 < 4x - 4 + 4$ $4x - 4 + 4 < 4 + 4$
 $-20 < 4x$ $4x < 8$

and: intersections \rightarrow overlap
 or: Union \rightarrow all

Practice & Problem Solving

38. **Construct Arguments** Describe and correct the error a student made graphing the compound inequality $x > 3$ or $x < -1$.



Solve each compound inequality and graph the solution.

- 39. $2x - 3 > 5$ or $3x - 1 < 8$
- 40. $x - 6 \leq 18$ and $3 - 2x \leq 11$
- 41. $\frac{1}{2}x - 5 > -3$ or $\frac{2}{3}x + 4 < 3$
- 42. $3(2x - 5) > 15$ and $4(2x - 1) > 10$

43. **Model With Mathematics** Lucy plans to spend between \$50 and \$65, inclusive, on

28) $xy = k$; y $\rightarrow y = \frac{k}{x}$

29) $a = \frac{2}{b} + 3c$; c
 $\frac{a - \frac{2}{b}}{3} = \frac{bc}{3}$

30) $6(2c + 3d) = 5(4c - 3d)$; d
 $12c + 18d = 20c - 15d$
 $-12c + 15d = -12c + 15d$
 $33d = 8c$
 $d = \frac{8c}{33}$

32) $x \leq -1$ or $x \geq 3$

33) $x + 8 > 11$
 $x > 3$ open

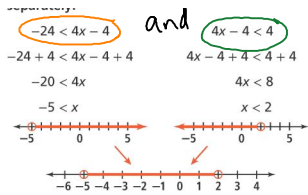
34) $4x + 3 \leq -6$
 $4x \leq -9$
 $x \leq -\frac{9}{4}$ closed

35) $2.4x - 9 < 1.8x + 6$
 $0.6x - 9 < 1.8x + 6$
 $0.6x < 15$
 $x < 25$ open

36) $3x - 8 \geq 4(x - 1.5)$
 $3x - 8 \geq 4x - 6$
 $-8 \geq x - 6$
 $-2 \geq x$
 $x \leq -2$ closed

39) $2x - 3 > 5$ or $3x - 1 < 8$
 $2x > 8$ $3x < 9$
 $x > 4$ or $x < 3$
 Union open

40) $x - 6 \leq 18$ and $3 - 2x \leq 11$
 $x \leq 24$ and $-2x \leq 8$
 $x \geq -4$
 Intersection closed



The solution is $x > -5$ and $x < 2$, or $-5 < x < 2$.

41. $\frac{1}{2}x - 5 > -3$ or $\frac{2}{3}x + 4 < 3$
 42. $3(2x - 5) > 15$ and $4(2x - 1) > 10$
 43. **Model With Mathematics** Lucy plans to spend between \$50 and \$65, inclusive, on packages of beads and packages of charms. If she buys 5 packages of beads at \$4.95 each, how many packages of charms at \$6.55 can Lucy buy while staying within her budget?

LESSON 1-7 Absolute Value Equations and Inequalities

Quick Review

When solving an equation or an inequality that contains an absolute value expression, you must consider both the positive and negative values of the absolute value expression.

Example

What is the value of x in $|4x + 7| < 43$?

Write and solve inequalities for the two cases.

$4x + 7$ is positive .	$4x + 7$ is negative .
$4x + 7 < 43$	$4x + 7 > -43$
$4x + 7 - 7 < 43 - 7$	$4x + 7 - 7 > -43 - 7$
$4x < 36$	$4x > -50$
$x < 9$	$x > -12.5$

The solution is $-12.5 < x < 9$.

Practice & Problem Solving

44. **Make Sense and Persevere** Thato is solving the absolute value equation $|3x| - 5 = 13$. What is the first step he should take?

Solve each absolute value equation or inequality.

45. $3 = x + 1$	46. $4 x - 5 = 24$
47. $3 > x - 6$	48. $ 2x - 3 \leq 12$

49. **Make Sense and Persevere** A person's normal body temperature is 98.6°F. According to physicians, a person's body temperature should not be more than 0.5°F from the normal temperature. How could you use an absolute value inequality to represent the temperatures that fall outside of normal range? Explain.