TOPIC 3

Topic Review

TOPIC ESSENTIAL QUESTION

1. How can linear functions be used to model situations and solve problems?

Vocabulary Review

Choose the correct term to complete each sentence.

2. When y tends to increase as x increases, the two data sets have a ________________.
3. A(n) ___________ is the difference between an actual and a predicted data value.
4. A(n) ___________ is an ordered list of numbers that often forms a pattern.
5. A trend line that most closely models the relationship between two variables displayed in a scatter plot is the ______________.
6. The ____________________ measures the strength and direction of the relationship between two variables in a linear model.

- arithmetic sequence
- correlation coefficient
- line of best fit
- linear regression
- positive association
- residual
- sequence
- term of a sequence
- trend line

Concepts & Skills Review

LESSON 3-1 Relations and Functions

Quick Review

A **function** is a relation in which every input, or element of the **domain**, is associated with exactly one output, or element of the **range**.

Example

Identify the domain and range of the ordered pairs in the table. Do the ordered pairs represent a function? Justify your response.

x	–2	0	3	5	9
y	–3.5	–1.5	1.5	3.5	7.5

The domain is {–2, 0, 3, 5, 9}. The range is {–3.5, –1.5, 1.5, 3.5, 7.5}.

Each element of the domain is associated with exactly one element of the range, so the ordered pairs represent a function.

Practice & Problem Solving

Identify the domain and range of each relation. Is the relation a function? Explain.

7. {(4, 1), (2, 3), (0, 4), (5, 3)}

8.

x	–1	–5	4	0	2
y	–5	–2	0	3	2

Reason **For 9 and 10, would a reasonable domain include all real numbers? Explain.**

9. A person drinks n ounces of a 20-ounce bottle of a sports drink.
10. A printer prints p pages at a rate of 25 pages per minute.

Construct Arguments **What constraints, if any, are there on the domain? Explain.**

11. An airplane ascends to a cruising altitude at the rate of 1,000 ft/min for m minutes.
12. The value of an automobile in d dollars decreases by about 10% each year.

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LESSON 3-2 Linear Functions

Quick Review

A **linear function** is a function whose graph is a straight line. It represents a linear relationship between two variables. A linear function written in **function notation** is $f(x) = mx + b$ and $f(x)$ is read "*f* of *x*."

Example

A taxi company charges \$3.50 plus \$0.85 per mile. What linear function can be used to determine the cost of a taxi ride of *x* miles? How much would a 3.5-mile taxi ride cost?

Let d = distance of the taxi ride.

Cost of taxi ride = cost × distance + fee

$$f(d) = 0.85d + 3.5$$

Use the function to determine the cost of a 3.5-mile ride.

$$f(3.5) = (0.85)(3.5) + 3.5$$
$$= 6.475$$

The cost of a 3.5-mile taxi ride is \$6.48.

Practice and Problem Solving

Evaluate each function for the elements in the domain {–4, –2, 0, 2, 4}.

13. $f(x) = 2x - 1$

14. $f(t) = -3(t - 2)$

15. Make Sense and Persevere Melissa runs a graphic design business. She charges by the page, and has a setup fee. The table shows her earnings for the last few projects. What is her per-page rate, and what is her setup fee?

Cost (\$)	185	335	485	635
Page totals	2	4	6	8

16. Use Structure Tia's Computer Repair Shop charges the labor rates shown for computer repairs. What linear function can she use to determine the cost of a repair that takes 5.5 hours and includes \$180 in parts?

Hours	1	1.5	2	2.5
Labor (\$)	85	127.5	170	212.5

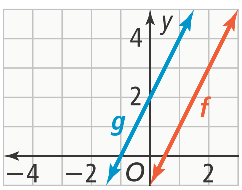
LESSON 3-3 Transforming Linear Functions

Quick Review

A **transformation** of a function *f* maps each point of its graph to a new location. A **translation** shifts each point of the graph of a function the same distance horizontally, vertically, or both. **Stretches** and **compressions** scale each point of a graph either horizontally or vertically.

Example

Let $f(x) = 2x - 1$. If $g(x) = (2x - 1) + 3$, how does the graph of *g* compare to the graph of *f*?



The graph of *g* is the translation of the graph of *f* three units up.

Practice & Problem Solving

Given the function $f(x) = x$, how does the addition or subtraction of a constant to the output affect the graph?

17. $f(x) = x - 2$

18. $f(x) = x + 5$

Given $f(x) = 4x - 5$, describe how the graph of *g* compares with the graph of *f*.

19. $g(x) = 4(x - 3) - 5$

20. $g(x) = 2(4x - 5)$

21. Reason Given $f(x) = -3x + 9$, how does multiplying the output of *f* by 2 affect the slope and *y*-intercept of the graph?

22. Model With Mathematics A hotel business center charges \$40 per hour to rent a computer plus a \$65 security deposit. The total rental charge is represented by $f(x) = 40x + 65$. How would the equation change if the business center increased the security deposit by \$15?

LESSON 3-4 Arithmetic Sequences

Quick Review

A **sequence** is an ordered list of numbers that often follows a pattern. Each number is a **term of the sequence**. In an **arithmetic sequence**, the difference between any two consecutive terms is a constant called the **common difference, *d***.

The **recursive formula** is used to describe the sequence and find the next term in a sequence from a given term. The **explicit formula** is used to find a specific term of the sequence.

Example

What is the 12th term in the sequence shown?

−8, −5.5, −3, −0.5, 2.0,…

Determine the recursive formula to describe the sequence.

The common difference, d, is 2.5.

$$a_1 = -8$$

$$a_n = a_{n-1} + 2.5$$

Use the explicit formula to find the 12th term.

$$a_{12} = -8 + (12 - 1)2.5$$

$$a_{12} = 19.5$$

The 12th term of the sequence is 19.5.

Practice & Problem Solving

Tell whether each sequence is an arithmetic sequence. If it is, give the common difference. If it is not, explain why.

23. 48, 45, 41, 38, 34,…

24. −6, 5, 16, 27, 38,…

Write a recursive formula for each arithmetic sequence.

25. 2, 6, 10, 14, 18,…

26. −5, −8.5, −12, −15.5, −19,…

27. **Reason** A table of data of an arithmetic sequence is shown. Use the explicit formula for the arithmetic sequence to find the 15th term.

x	1	2	3	4	5
y	8	13	18	23	28

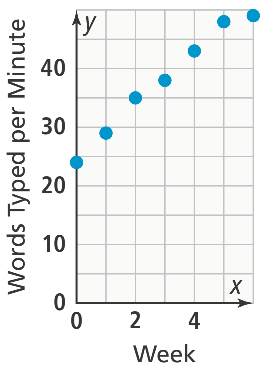
28. **Make Sense and Persevere** Gabriela is selling friendship bracelets for a school fundraiser. After the first day, she has 234 bracelets left. After the second day, she has 222 left. Assuming the sales pattern continues and it is an arithmetic sequence, how many bracelets will Gabriela have left to sell after the fifth day?

Quick Review

A paired data set (or a bivariate data set) has a **positive association** when *y*-values tend to increase as *x*-values increase and a **negative association** when *y*-values tend to decrease as *x*-values increase. When the relationship between the paired data can be modeled with a linear function, a **line of best fit** represents the relationship. The paired data are positively correlated if *y*-values increase as *x*-values increase and negatively correlated if *y*-values decrease as *x*-values increase.

Example

Yama takes a course to improve his typing speed. The scatter plot shows his progress over six weeks. What is the relationship between the number of weeks and Yama's typing speed?

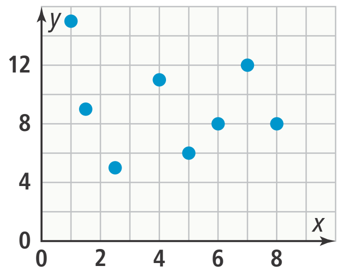


As the number of weeks of practice increase, so does the number of words typed per minute. The scatter plot shows a positive association that is approximately linear suggesting a positive correlation between weeks and words typed per minute.

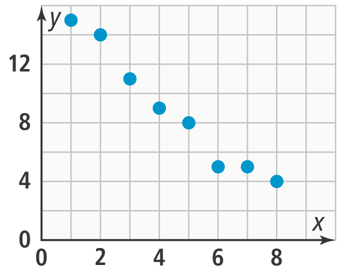
Practice & Problem Solving

Describe the type of association each scatter plot shows.

29.



30.



31. **Reason** Where should the line of best fit be in relationship to the points plotted on a scatter plot?

Use Appropriate Tools **For each table, make a scatter plot of the data. If the data suggest a linear relationship, draw a trend line and write its equation.**

32.

x	*y*
2	5
4	8
6	12
8	14
10	18

33.

x	*y*
3	40
6	36
8	31
12	27
15	24

34. **Model With Mathematics** The table shows the recommended distance of a light source *y* in feet, from the wall for different ceiling heights *x* in feet. What is the equation of the trend line that models the data shown in the table? What does the slope of the trend line represent?

x	8	9	10	11	12
y	20	27	33	40	48

LESSON 3-6 Analyzing Lines of Fit

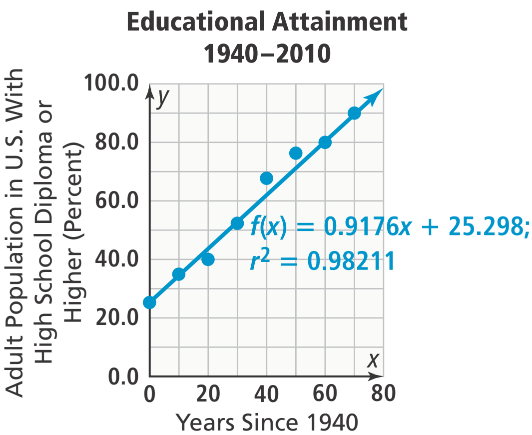
Quick Review

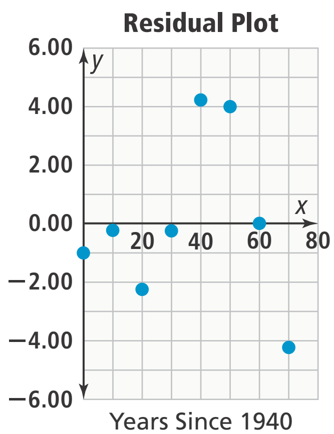
Linear regression is a method used to calculate line of best fit. The **correlation coefficient** indicates the direction and strength of the linear relationship between two variables.

A **residual** reveals how well a linear model fits the data set. You can use the line of best fit to estimate a value within a range of known values (**interpolation**) or predict a value outside the range of known values (**extrapolation**).

Example

The scatter plot shows the percentage of American adults with a high school diploma or higher from 1940 to 2010. Based on the residual plot below the scatter plot, how appropriate is the linear model for the data?





The residual plot shows the residuals distributed above and below the x-axis and clustered somewhat close to the x-axis. The linear model is likely a good fit for the data.

Practice & Problem Solving

Use technology to perform a linear regression to determine the equation for the line of best fit for the data. Estimate the value of y when $x = 25$.

35.

x	y
20	9
22	12
24	16
26	20
28	23

36.

x	y
6	85
12	81
18	75
24	69
30	63

37. **Reason** How are interpolation and extrapolation similar? How are they different?

38. **Model With Mathematics** The table shows the winning times for the 100-meter run in the Olympics since 1928. What is the equation of the line of best fit for the data? What do the slope and y-intercept represent? Estimate the winning time in 2010, and predict the winning time in 2020.

Year	Time (s)	Year	Time (s)
1928	10.80	1980	10.25
1932	10.30	1984	9.99
1936	10.30	1988	9.92
1948	10.30	1992	9.96
1952	10.40	1996	9.84
1956	10.50	2000	9.87
1960	10.20	2004	9.85
1964	10.00	2008	9.69
1968	9.95	2012	9.63
1972	10.14	2016	9.81
1976	10.06		

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